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An Enhanced Spherical Cubic fuzzy WASPAS Method and its Application for the Assessment of Service Quality of Crowdsourcing

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ABSTRACT

The service quality of crowdsourcing logistics platforms (CLP) represents the fulfillment of standardized requirements during service delivery. Evaluating this quality enables quantification of service performance, identification of deficiencies in key indicators and optimization of operational efficiency. Such assessment helps standardize courier behavior, enhance user satisfaction, and strengthen platform competitiveness. This study proposes an enhanced decision-making method for assessing CLP service quality under uncertainty using spherical cubic fuzzy (SCF) sets, which effectively capture fuzzy and uncertain information. We develop an SCF-based approach integrating Renyi entropy and the WASPAS method, featuring four key contributions: (1) definition of SCF Aczel-Alsina operations and four corresponding aggregation operators (weighted averaging, geometric, ordered weighted averaging, and ordered weighted geometric); (2) development of an SCF-Renyi entropy weight method using score functions to determine criteria weights; (3) improvement of the WASPAS method through SCF Aczel-Alsina operators; and (4) validation through a supplier selection case study demonstrating the method's applicability. Sensitivity analysis confirms the approach's stability and universality in handling multi-criteria decision analysis problems.

1. Introduction

With the remarkable advancement of internet technologies, the rapid emergence of e-commerce, fresh produce delivery, and food delivery services has driven explosive growth in time-sensitive logistics demands. Traditional logistics systems struggle to meet modern high-standard requirements due to inefficiencies and declining service quality. Furthermore, as the sharing economy matures and mobile internet technologies evolve rapidly, critical challenges in enhancing logistics service quality lie in comprehensively integrating data matching technologies, route planning optimization, and dynamic adjustments. In this context, crowdsourcing logistics services have emerged to address the shortcomings of traditional logistics systems. As an innovative integration of

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the sharing economy and logistics industry, crowdsourcing logistics leverages internet platforms to consolidate decentralized social resources (e.g., individuals, vehicles, and idle labor) through crowdsourcing models to fulfill delivery tasks. The rise of crowdsourcing logistics services demonstrates significant advantages and social value: (1) It reduces enterprises' fixed asset investments, enabling asset-light operations; (2) It enhances environmental sustainability by lowering carbon emissions; (3) It accelerates logistics industry digitalization and drives business model innovation. Currently, the development of crowdsourcing logistics platforms is gaining momentum, yet the absence of robust institutional frameworks hinders the identification of platforms with competitive potential. Given that platform selection constitutes a multi-criteria decision-making (MCDM) problem under uncertain environments, this study establishes a decision-making model under uncertain environments to support the evaluation and selection of crowdsourcing logistics service platforms.

Multi-Criteria Decision Analysis (MCDA), a pivotal branch of decision science, involves the process of evaluating and prioritizing alternative solutions based on predefined criteria, incorporating decision-makers’ cognitive preferences and assessments. The theories and methodologies of MCDA have been extensively applied in domains such as supply chain management, energy risk evaluation, and digital transformation. However, due to the inherent fuzziness and uncertainty of decision-makers’ cognitive capabilities, precise numerical expressions often prove inadequate for capturing their indeterminate perceptions, particularly when evaluating qualitative indicators. Consequently, a series of extended fuzzy set theories rooted in Zadeh’s foundational fuzzy set theory have been proposed to characterize uncertain and ambiguous information [1]. These include intuitionistic fuzzy sets [2], Pythagorean fuzzy sets [3, 4], q-rung orthopair fuzzy sets [5], Fermatean fuzzy sets [6], and their extended theoretical frameworks [7-9]. These fuzzy sets have garnered significant scholarly attention globally, yielding prolific research outcomes in both theoretical explorations and practical applications, thereby facilitating the widespread adoption of fuzzy set-based decision-making methodologies across diverse fields [10-14]. Nevertheless, limitations persist in these frameworks when addressing imprecise and inconsistent information. To address this, scholars such as Coung [15] introduced the concept of picture fuzzy sets, which delineate uncertain information through membership, non-membership, and hesitancy degrees. Subsequently, Mahmood *et al.,* [16] expanded the representational space of fuzzy sets by proposing spherical fuzzy sets. This advancement spurred extensive research into decision-making theories under spherical fuzzy environments, particularly in areas such as information measures [17, 18], aggregation operators [19, 20], and decision methodologies. [21-23]. Recently, SCF sets, a hybrid framework integrating spherical fuzzy sets and cubic sets, have been developed to enhance the characterization of uncertain information. This innovation leverages cubic sets to represent the membership, non-membership, and indeterminacy degrees inherent in spherical fuzzy sets, thereby augmenting their capacity to model complex uncertainty. Based on this novel extension of spherical fuzzy set, Ayaz *et al.,* [24] proposed a family of Hamacher aggregation operators with SCF information to evaluate enterprise production. Ali *et al.,* [25] propounded an extension of ELiminating Et Choice Translating REality (ELECTRE) method based on SCF theories. Mohammed *et al.,* [26] proposed a SCF decision methodology including the fuzzy decision by opinion score method (FDOSM) and the fuzzy weighted with zero inconsistency criterion (FWZIC) method to determining the superiority of a robust cloud fault tolerance mechanism. However, there is no study to investigate the WASPAS method based on Aczel-Alsina operator under SCF setting.

The WASPAS (Weighted Aggregated Sum Product Assessment) method is distinguished by its innovative dual-model integration framework, which synthesizes the outcomes of the Weighted Sum Model (WSM) and Weighted Product Model (WPM) through linear aggregation to achieve comprehensive multi-criteria evaluation of alternatives [27]. This methodology offers three principal advantages: (1) By integrating the complementary strengths of WSM and WPM, it effectively mitigates potential biases inherent in individual approaches, thereby enhancing the reliability of final decision outcomes. (2) It possesses the capability to simultaneously accommodate both benefit-type (maximization) and cost-type (minimization) criteria, facilitating structured analysis of complex decision-making problems. (3) The procedural simplicity of the method, requiring no complex iterative computations, renders it particularly suitable for large-scale or real-time decision scenarios. In light of the mentioned merits of WASPAS method, several extensions of WASPAS in diverse fuzzy sets are proposed for real decision analysis [28-32]. In the existing literature, the WASPAS method is improved by different aggregation operators to deal with dissimilar kinds of practical decision problems. However, there is no literature showing that the WASPAS method has been studied in SCF set.

Based on the mentioned discussion and literature overview, the principal contributions of this study are systematically delineated as follows:

i. Formulation of SCF Aczel-Alsina operational rules through rigorous integration of Aczel-Alsina normative principles, establishing a novel computational framework for SCF (SCF) information processing;
ii. Development of four innovative SCF aggregation operators underpinned by Aczel-Alsina norms, including SCF Aczel-Alsina weighted averaging (SCFAAWA) operator, SCF Aczel-Alsina ordered weighted averaging (SCFAAOWA) operator, SCF Aczel-Alsina weighted geometric (SCFAAWG) operator, SCF Aczel-Alsina ordered weighted geometric (SCFAAOWG) operator;
iii. Proposition of a hybrid decision-weighting methodology combining SCF-Rényi entropy measures with score function analysis, enabling robust determination of criterion weights in uncertain environments;
iv. Establishment of an enhanced WASPAS (Weighted Aggregated Sum Product Assessment) decision framework synergistically integrating the developed SCF Aczel-Alsina aggregation operators, facilitating comprehensive multi-criteria evaluation under SCF uncertainty.

The architectural organization of this research is structured as follows: Section 2 revisits essential definitions and fundamental concepts related to SCF sets. Section 3 rigorously develops the theoretical foundations for Aczel-Alsina aggregation operators within SCF environments. Section 4 elaborates the integrated decision-making framework combining the novel aggregation operators, Rényi entropy weighting, and WASPAS methodology. Section 5 empirically validates the proposed approach through comprehensive case analyses encompassing applicability assessment, sensitivity examination, and practical implementation studies. Section 6 synthesizes the research outcomes and delineates prospective research trajectories.

2. Fundamental conceptions of SCFSs

The current part recalls several fundamental preliminaries including cubic sets, SFSs and SCF sets. In addition, some basic notions of SCF sets are introduced to construct the enhanced decision approach. In the full paper, Δ is defined as a universal set.

Definition 1 [33]. A cubic set Φ on Δ is defined as below:

$$\Phi = \left\{\left\langle x, \tilde{\Omega}_{\Phi}(x), \tilde{\Xi}_{\Phi}(x)\right\rangle | x \in \Delta\right\}, \tag{1}$$

where $\tilde{\Omega}_{\Phi}: \Delta \to P[0,1]$ and $\tilde{\Xi}_{\Phi}: \Delta \to [0,1]$ denote the membership and nonmembership grades, respectively.

Definition 2 [16]. A spherical fuzzy set Λ on Δ is defined as below:

$$\Lambda = \{\langle x, \alpha_{\Lambda}(x), \beta_{\Lambda}(x), \chi_{\Lambda}(x)\rangle | x \in \Delta\}, \tag{2}$$

wherein $\alpha_\Lambda(x), \beta_\Lambda(x), \chi_\Lambda(x): \Delta \to [0,1]$ indicate the positive, neutral, and negative membership function of an objective $x \in \Delta$ to Λ, respectively. The triple $(\alpha_\Lambda(x), \beta_\Lambda(x), \chi_\Lambda(x))$ meets the presupposition:$0 \le (\alpha_\Lambda(x))^2 + (\beta_\Lambda(x))^2 + (\chi_\Lambda(x))^2 \le 1$.

Definition 3 [24]. A SCF set Θ on Δ is articulated as below:

$$\Theta = \{\langle x, Q_\Theta(x), W_\Theta(x), E_\Theta(x)\rangle | x \in \Delta\}, \tag{3}$$

wherein $Q_\Theta(x) = ([\delta^-, \delta^+], \delta)$, $W_\Theta(x) = ([\varepsilon^-, \varepsilon^+], \varepsilon)$ and $E_\Theta(x) = ([\phi^-, \phi^+], \phi)$ indicate the positive, neutral, and negative membership grades of an objective $x \in \Delta$ to Λ respectively, which satisfies the conditions that $0 \le (sup([\delta^-, \delta^+]))^2 + (sup([\varepsilon^-, \varepsilon^+]))^2 + (sup([\phi^-, \phi^+]))^2 \le 1$ and $0 \le \delta^2 + \varepsilon^2 + \phi^2 \le 1$. The hesitant grade of SCF set is defined as $\pi_\Theta(x) = \left(\sqrt{1 - ((sup([\delta^-, \delta^+]))^2 + (sup([\varepsilon^-, \varepsilon^+]))^2 + (sup([\phi^-, \phi^+]))^2)}, \sqrt{1 - (\delta^2 + \varepsilon^2 + \phi^2)}\right)$. For convenience, a SCF number (SCFN) is depicted as: $\Theta = (Q_\Theta(x), W_\Theta(x), E_\Theta(x)) = \{([\delta^-, \delta^+], \delta), ([\varepsilon^-, \varepsilon^+], \varepsilon), ([\phi^-, \phi^+], \phi)\}$.

Definition 4 [24]. For three SCFNs $\Theta = \{([\delta^-, \delta^+], \delta), ([\varepsilon^-, \varepsilon^+], \varepsilon), ([\phi^-, \phi^+], \phi)\}$, $\Theta_j = \{([\delta_j^-, \delta_j^+], \delta_j), ([\varepsilon_j^-, \varepsilon_j^+], \varepsilon_j), ([\phi_j^-, \phi_j^+], \phi_j)\} (j = 1,2)$ and a real number λ, the operations of SCFNs are depicted as:

(1) $\Theta_1 \oplus \Theta_2 = \left\{\left(\begin{matrix}\left[\begin{matrix}\sqrt{(\delta_1^-)^2 + (\delta_2^-)^2 - (\delta_1^-)^2(\delta_2^-)^2}, \\ \sqrt{(\delta_1^+)^2 + (\delta_2^+)^2 - (\delta_1^+)^2(\delta_2^+)^2}\end{matrix}\right], \\ \sqrt{(\delta_1)^2 + (\delta_2)^2 - (\delta_1)^2(\delta_2)^2}\end{matrix}\right), \left(\begin{matrix}\left[\begin{matrix}\varepsilon_1^-\varepsilon_2^-, \\ \varepsilon_1^+\varepsilon_2^+\end{matrix}\right], \\ \varepsilon_1\varepsilon_2\end{matrix}\right), \left(\begin{matrix}\left[\begin{matrix}\phi_1^-\phi_2^-, \\ \phi_1^+\phi_2^+\end{matrix}\right], \\ \phi_1\phi_2\end{matrix}\right)\right\}$;

(2) $\Theta_1 \otimes \Theta_2 = \left\{\left(\begin{matrix}\left[\begin{matrix}\delta_1^-\delta_2^-, \\ \delta_1^+\delta_2^+\end{matrix}\right], \\ \delta_1\delta_2\end{matrix}\right), \left(\begin{matrix}\left[\begin{matrix}\varepsilon_1^-\varepsilon_2^-, \\ \varepsilon_1^+\varepsilon_2^+\end{matrix}\right], \\ \varepsilon_1\varepsilon_2\end{matrix}\right), \left(\begin{matrix}\left[\begin{matrix}\sqrt{(\phi_1^-)^2 + (\phi_2^-)^2 - (\phi_1^-)^2(\phi_2^-)^2}, \\ \sqrt{(\phi_1^+)^2 + (\phi_2^+)^2 - (\phi_1^+)^2(\phi_2^+)^2}\end{matrix}\right], \\ \sqrt{(\phi_1)^2 + (\phi_2)^2 - (\phi_1)^2(\phi_2)^2}\end{matrix}\right)\right\}$;

(3) $\lambda\Theta = \left\{\begin{matrix}\left(\sqrt{1 - (1 - (\delta^-)^2)^\lambda}, \sqrt{1 - (1 - (\delta^+)^2)^\lambda}, \sqrt{1 - (1 - (\delta)^2)^\lambda}\right), \\ \left([(\varepsilon^-)^\lambda, (\varepsilon^+)^\lambda], (\varepsilon)^\lambda\right), \left([(\phi^-)^\lambda, (\phi^+)^\lambda], (\phi)^\lambda\right)\end{matrix}\right\}$;

(4) $(\Theta)^\tau = \left\{\begin{matrix}([(\delta^-)^\tau, (\delta^+)^\tau], (\delta)^\tau), ([(\varepsilon^-)^\tau, (\varepsilon^+)^\tau], (\varepsilon)^\tau), \\ \left(\sqrt{1 - (1 - (\phi^-)^2)^\tau}, \sqrt{1 - (1 - (\phi^+)^2)^\tau}, \sqrt{1 - (1 - (\phi)^2)^\tau}\right)\end{matrix}\right\}$.

Definition 5 [24]. For a SCFN $\Theta = \{([\delta^-, \delta^+], \delta), ([\varepsilon^-, \varepsilon^+], \varepsilon), ([\phi^-, \phi^+], \phi)\}$, then the score function of Θ is defined as:

$$\tilde{S}(\Theta) = \frac{1}{2}\left(1 + \frac{1}{9}((\delta^- + \delta^+ + \delta)^2 + (\varepsilon^- + \varepsilon^+ + \varepsilon)^2 - (\phi^- + \phi^+ + \phi)^2)\right). \tag{4}$$

Definition 6 [24]. For a SCFN $\Theta = \{([\delta^-, \delta^+], \delta), ([\varepsilon^-, \varepsilon^+], \varepsilon), ([\phi^-, \phi^+], \phi)\}$, then the accuracy function of Θ is defined as:

$$\tilde{A}(\Theta) = \frac{1}{9}((\delta^- + \delta^+ + \delta)^2 + (\varepsilon^- + \varepsilon^+ + \varepsilon)^2 + (\phi^- + \phi^+ + \phi)^2). \tag{5}$$

Definition 7 [24]. For two SCFNs $\Theta_j = \{([\delta_j^-, \delta_j^+], \delta_j), ([\varepsilon_j^-, \varepsilon_j^+], \varepsilon_j), ([\phi_j^-, \phi_j^+], \phi_j)\} (j = 1,2)$, the comparison laws are given as:

If $\tilde{S}(\Theta_1) > \tilde{S}(\Theta_2)$, then $\Theta_1 \succ \Theta_2$;
If $\tilde{S}(\Theta_1) < \tilde{S}(\Theta_2)$, then $\Theta_1 \prec \Theta_2$;
If $\tilde{S}(\Theta_1) = \tilde{S}(\Theta_2)$, then
    If $\tilde{A}(\Theta_1) > \tilde{A}(\Theta_2)$, then $\Theta_1 \succ \Theta_2$;
    If $\tilde{A}(\Theta_1) < \tilde{A}(\Theta_2)$, then $\Theta_1 \prec \Theta_2$;

If $\tilde{A}(\Theta_1) = \tilde{A}(\Theta_2)$, then $\Theta_1 \sim \Theta_2$.

Definition 8 [24]. Let $\Theta_j = \{([\delta_j^-, \delta_j^+], \delta_j), ([\varepsilon_j^-, \varepsilon_j^+], \varepsilon_j), ([\phi_j^-, \phi_j^+], \phi_j)\}(j = 1,2)$ be a set of SCFNs. where ϖ_j is the weight of SCFN Θ_j with $\sum_{j=1}^{n} \varpi_j = 1, \varpi_j \in [0,1]$. Then SCF weighted averaging operator (SCFWA) operator is depicted as:

$$SCFWA(\Theta_1, \Theta_2, \ldots, \Theta_n) = \overset{j=1}{\underset{n}{\oplus}} \varpi_j \Theta_j$$

$$= \left\{ \left(\begin{bmatrix} \sqrt{1 - \prod_{j=1}^{n} \left(1 - (\delta_j^-)^2\right)^{\varpi_j}}, \\ \sqrt{1 - \prod_{j=1}^{n} \left(1 - (\delta_j^+)^2\right)^{\varpi_j}} \end{bmatrix}, \\ \sqrt{1 - \prod_{j=1}^{n} \left(1 - (\delta_j)^2\right)^{\varpi_j}} \right), \left(\begin{bmatrix} \prod_{j=1}^{n} (\varepsilon_j^-)^{\varpi_j}, \\ \prod_{j=1}^{n} (\varepsilon_j^+)^{\varpi_j} \end{bmatrix}, \\ \prod_{j=1}^{n} (\varepsilon_j)^{\varpi_j} \right), \left(\begin{bmatrix} \prod_{j=1}^{n} (\phi_j^-)^{\varpi_j}, \\ \prod_{j=1}^{n} (\phi_j^+)^{\varpi_j} \end{bmatrix}, \\ \prod_{j=1}^{n} (\gamma_j)^{\varpi_j} \right) \right\} \tag{6}$$

Definition 9 [24]. Let $\Theta_j = \{([\delta_j^-, \delta_j^+], \delta_j), ([\varepsilon_j^-, \varepsilon_j^+], \varepsilon_j), ([\phi_j^-, \phi_j^+], \phi_j)\}(j = 1,2)$ be a set of SCFNs. where ϖ_j is the weight of SCFN Θ_j with $\sum_{j=1}^{n} \varpi_j = 1, \varpi_j \in [0,1]$. Then SCF weighted geometric operator (SCFWG) operator is depicted as:

$$SCFWG(\Theta_1, \Theta_2, \ldots, \Theta_n) = \overset{j=1}{\underset{n}{\otimes}} (\Theta_j)^{\varpi_j}$$

$$= \left\{ \left(\begin{bmatrix} \prod_{j=1}^{n} (\delta_j^-)^{\varpi_j}, \\ \prod_{j=1}^{n} (\delta_j^+)^{\varpi_j} \end{bmatrix}, \\ \prod_{j=1}^{n} (\delta_j)^{\varpi_j} \right), \left(\begin{bmatrix} \prod_{j=1}^{n} (\varepsilon_j^-)^{\varpi_j}, \\ \prod_{j=1}^{n} (\varepsilon_j^+)^{\varpi_j} \end{bmatrix}, \\ \prod_{j=1}^{n} (\varepsilon_j)^{\varpi_j} \right), \left(\begin{bmatrix} \sqrt{1 - \prod_{j=1}^{n} \left(1 - (\phi_j^-)^2\right)^{\varpi_j}}, \\ \sqrt{1 - \prod_{j=1}^{n} \left(1 - (\phi_j^+)^2\right)^{\varpi_j}} \end{bmatrix}, \\ \sqrt{1 - \prod_{j=1}^{n} \left(1 - (\phi_j)^2\right)^{\varpi_j}} \right) \right\}. \tag{7}$$

3. Some novel SCF Aczel–Alsina operators

Information aggregation is a vital tool to fuse the fuzzy assessment information provided by multiple experts. As the part of Archimedean t-norms and t-conorms group, Aczel–Alsina norms have excellent property and more flexibility than algebraic norms. Due to the notion of Aczel–Alsina sum and product operations, we define the Aczel–Alsina rules for SCFNs and further propound SCF operators to aggregate the evaluation information in the environment of SCF sets.

Definition 10 [34]. Let a and b are two arbitrary, non-negative real numbers$(a, b > 0)$, then the definition of Aczel–Alsina norms can be described as:

$$T_{AA}^{\partial}(a, b) = exp\left\{-\left((-\ln a)^{\partial} + (-\ln b)^{\partial}\right)^{\frac{1}{\partial}}\right\}, \quad \partial > 0, \tag{8}$$

$$S_{AA}^{\partial}(a, b) = 1 - exp\left\{-\left((-\ln(1-a))^{\partial} + (-\ln(1-b))^{\partial}\right)^{\frac{1}{\partial}}\right\}, \quad \partial > 0. \tag{9}$$

Definition 11. For three SCFNs $\Theta = \{([\delta^-, \delta^+], \delta), ([\varepsilon^-, \varepsilon^+], \varepsilon), ([\phi^-, \phi^+], \phi)\}$, $\Theta_j = \{([\delta_j^-, \delta_j^+], \delta_j), ([\varepsilon_j^-, \varepsilon_j^+], \varepsilon_j), ([\phi_j^-, \phi_j^+], \phi_j)\}(j = 1,2)$ and a real number λ, the SCF Aczel–Alsina operations are defined as:

$$\Theta_1 \oplus \Theta_2 = \left\{ \begin{array}{l} \left(\left[\begin{array}{l} \sqrt{1 - exp\left\{-((-ln(1-(\delta_1^-)^2))^\partial + (-ln(1-(\delta_2^-)^2))^\partial)^{\frac{1}{\partial}}\right\}}, \\ \sqrt{1 - exp\left\{-((-ln(1-(\delta_1^+)^2))^\partial + (-ln(1-(\delta_2^+)^2))^\partial)^{\frac{1}{\partial}}\right\}} \end{array} \right], \\ \sqrt{1 - exp\left\{-((-ln(1-(\delta_1)^2))^\partial + (-ln(1-(\delta_2)^2))^\partial)^{\frac{1}{\partial}}\right\}} \end{array} \right), \\ \left(\left[\begin{array}{l} \sqrt{exp\left\{-((-ln((\varepsilon_1^-)^2))^\partial + (-ln((\varepsilon_2^-)^2))^\partial)^{\frac{1}{\partial}}\right\}}, \\ \sqrt{exp\left\{-((-ln((\varepsilon_1^+)^2))^\partial + (-ln((\varepsilon_2^+)^2))^\partial)^{\frac{1}{\partial}}\right\}} \end{array} \right], \\ \sqrt{exp\left\{-((-ln((\varepsilon_1)^2))^\partial + (-ln((\varepsilon_2)^2))^\partial)^{\frac{1}{\partial}}\right\}} \end{array} \right), \\ \left(\left[\begin{array}{l} \sqrt{exp\left\{-((-ln((\phi_1^-)^2))^\partial + (-ln((\phi_2^-)^2))^\partial)^{\frac{1}{\partial}}\right\}}, \\ \sqrt{exp\left\{-((-ln((\phi_1^+)^2))^\partial + (-ln((\phi_2^+)^2))^\partial)^{\frac{1}{\partial}}\right\}} \end{array} \right] \\ \sqrt{exp\left\{-((-ln((\phi_1)^2))^\partial + (-ln((\phi_2)^2))^\partial)^{\frac{1}{\partial}}\right\}} \end{array} \right) \end{array} \right\};$$

$$\Theta_1 \otimes \Theta_2 = \left\{ \begin{array}{l} \left(\left[\begin{array}{l} \sqrt{exp\left\{-((-ln((\delta_1^-)^2))^\partial + (-ln((\delta_2^-)^2))^\partial)^{\frac{1}{\partial}}\right\}}, \\ \sqrt{exp\left\{-((-ln((\delta_1^+)^2))^\partial + (-ln((\delta_2^+)^2))^\partial)^{\frac{1}{\partial}}\right\}} \end{array} \right], \\ \sqrt{exp\left\{-((-ln((\delta_1)^2))^\partial + (-ln((\delta_2)^2))^\partial)^{\frac{1}{\partial}}\right\}} \end{array} \right), \\ \left(\left[\begin{array}{l} \sqrt{exp\left\{-((-ln((\varepsilon_1^-)^2))^\partial + (-ln((\varepsilon_2^-)^2))^\partial)^{\frac{1}{\partial}}\right\}}, \\ \sqrt{exp\left\{-((-ln((\varepsilon_1^+)^2))^\partial + (-ln((\varepsilon_2^+)^2))^\partial)^{\frac{1}{\partial}}\right\}} \end{array} \right], \\ \sqrt{exp\left\{-((-ln((\varepsilon_1)^2))^\partial + (-ln((\varepsilon_2)^2))^\partial)^{\frac{1}{\partial}}\right\}} \end{array} \right), \\ \left(\left[\begin{array}{l} \sqrt{1 - exp\left\{-((-ln(1-(\phi_1^-)^2))^\partial + (-ln(1-(\phi_2^-)^2))^\partial)^{\frac{1}{\partial}}\right\}}, \\ \sqrt{1 - exp\left\{-((-ln(1-(\phi_1^+)^2))^\partial + (-ln(1-(\phi_2^+)^2))^\partial)^{\frac{1}{\partial}}\right\}} \end{array} \right], \\ \sqrt{1 - exp\left\{-((-ln(1-(\phi_1)^2))^\partial + (-ln(1-(\phi_2)^2))^\partial)^{\frac{1}{\partial}}\right\}} \end{array} \right) \end{array} \right\};$$

$$\lambda\Theta=\left\{\begin{array}{c}\left(\left[\begin{array}{l}\sqrt{1-exp\left\{-(\lambda(-ln(1-(\delta^{-})^{2}))^{\partial})^{\frac{1}{\partial}}\right\}},\\ \sqrt{1-exp\left\{-(\lambda(-ln(1-(\delta^{+})^{2}))^{\partial})^{\frac{1}{\partial}}\right\}}\end{array}\right],\right.\\ \left.\sqrt{1-exp\left\{-(\lambda(-ln(1-(\delta)^{2}))^{\partial})^{\frac{1}{\partial}}\right\}}\right),\\ \left(\left[\begin{array}{l}\sqrt{exp\left\{-(\lambda(-ln((\varepsilon^{-})^{2}))^{\partial})^{\frac{1}{\partial}}\right\}},\\ \sqrt{exp\left\{-(\lambda(-ln((\varepsilon^{+})^{2}))^{\partial})^{\frac{1}{\partial}}\right\}}\end{array}\right],\right.\\ \left.\sqrt{1-exp\left\{-(\lambda(-ln((\varepsilon)^{2}))^{\partial})^{\frac{1}{\partial}}\right\}}\right),\\ \left(\left[\begin{array}{l}\sqrt{exp\left\{-(\lambda(-ln((\phi^{-})^{2}))^{\partial})^{\frac{1}{\partial}}\right\}},\\ \sqrt{exp\left\{-(\lambda(-ln((\phi^{+})^{2}))^{\partial})^{\frac{1}{\partial}}\right\}}\end{array}\right],\right.\\ \left.\sqrt{exp\left\{-(\lambda(-ln((\phi)^{2}))^{\partial})^{\frac{1}{\partial}}\right\}}\right)\end{array}\right\};$$

$$(\Theta)^{\lambda}=\left\{\begin{array}{c}\left(\left[\begin{array}{l}\sqrt{exp\left\{-(\lambda(-ln((\delta^{-})^{2}))^{\partial})^{\frac{1}{\partial}}\right\}},\\ \sqrt{exp\left\{-(\lambda(-ln((\delta^{+})^{2}))^{\partial})^{\frac{1}{\partial}}\right\}}\end{array}\right],\right.\\ \left.\sqrt{exp\left\{-(\lambda(-ln((\delta)^{2}))^{\partial})^{\frac{1}{\partial}}\right\}}\right),\\ \left(\left[\begin{array}{l}\sqrt{exp\left\{-(\lambda(-ln((\varepsilon^{-})^{2}))^{\partial})^{\frac{1}{\partial}}\right\}},\\ \sqrt{exp\left\{-(\lambda(-ln((\varepsilon^{+})^{2}))^{\partial})^{\frac{1}{\partial}}\right\}}\end{array}\right],\right.\\ \left.\sqrt{1-exp\left\{-(\lambda(-ln((\varepsilon)^{2}))^{\partial})^{\frac{1}{\partial}}\right\}}\right),\\ \left(\left[\begin{array}{l}\sqrt{1-exp\left\{-(\lambda(-ln(1-(\phi^{-})^{2}))^{\partial})^{\frac{1}{\partial}}\right\}},\\ \sqrt{1-exp\left\{-(\lambda(-ln(1-(\phi^{+})^{2}))^{\partial})^{\frac{1}{\partial}}\right\}}\end{array}\right],\right.\\ \left.\sqrt{1-exp\left\{-(\lambda(-ln(1-(\phi)^{2}))^{\partial})^{\frac{1}{\partial}}\right\}}\right)\end{array}\right\}.$$

Theorem 1. Let $\Theta_j=\{([\delta_j^-,\delta_j^+],\delta_j),([\varepsilon_j^-,\varepsilon_j^+],\varepsilon_j),([\phi_j^-,\phi_j^+],\phi_j)\}(j=1,2)$ be two SCFNs and $\lambda,\lambda_1,\lambda_2\geq 0$. Then, the following properties can be obtained:

(1) $\Theta_1\oplus\Theta_2=\Theta_2\oplus\Theta_1$.

(2) $\Theta_1\otimes\Theta_2=\Theta_2\otimes\Theta_1$.

(3) $\lambda(\Theta_1\oplus\Theta_2)=\lambda\Theta_1\oplus\lambda\Theta_2$.

(4) $\lambda_1\Theta_1 \oplus \lambda_2\Theta_1 = (\lambda_1 + \lambda_2)\Theta_1$.
(5) $(\Theta_1 \otimes \Theta_2)^{\lambda} = (\Theta_1)^{\lambda} \otimes (\Theta_2)^{\lambda}$.
(6) $(\Theta_1)^{\lambda_1} \otimes (\Theta_1)^{\lambda_2} = (\Theta_1)^{\lambda_1+\lambda_2}$.
Proof. It is trivial via Definition 11.

3.1 SCF Aczel–Alsina weighted averaging operator

The current section develops SCF Aczel–Alsina weighted and ordered weighted averaging operators utilizing the Aczel–Alsina norms.

Definition 12. Let $\Theta_{\lambda} = \{([\delta_{\lambda}^-, \delta_{\lambda}^+], \delta_{\lambda}), ([\varepsilon_{\lambda}^-, \varepsilon_{\lambda}^+], \varepsilon_{\lambda}), ([\phi_{\lambda}^-, \phi_{\lambda}^+], \phi_{\lambda})\}(\lambda = 1,2,\cdots,n)$ be a set of SCFNs. where ϖ_{λ} is the importance of SCFN Θ_{λ} with $\sum_{\lambda=1}^{n} \varpi_{\lambda} = 1, \varpi_{\lambda} \in [0,1]$. Then SCF Aczel–Alsina weighted averaging (SCFAAWA) operator is depicted as:

$$SCFAAWA(\Theta_1, \Theta_2, \ldots, \Theta_n) = \varpi_1\Theta_1 \oplus \varpi_2\Theta_2 \oplus \cdots \oplus \varpi_n\Theta_n. \tag{10}$$

Theorem 2. Let $\Theta_{\lambda} = \{([\delta_{\lambda}^-, \delta_{\lambda}^+], \delta_{\lambda}), ([\varepsilon_{\lambda}^-, \varepsilon_{\lambda}^+], \varepsilon_{\lambda}), ([\phi_{\lambda}^-, \phi_{\lambda}^+], \phi_{\lambda})\}(\lambda = 1,2,\cdots,n)$ be a set of SCFNs. Then the consolidated outcome of them by SCFAAWA operator is a SCFNs and represented as:

$$SCFAAWA(\Theta_1, \Theta_2, \ldots, \Theta_n) = \left\{ \begin{array}{c} \left(\left[\begin{array}{c} \sqrt{1 - exp\left\{-\left(\sum_{\lambda=1}^{n} \varpi_{\lambda} \left(-ln(1 - (\delta_{\lambda}^-)^2)\right)^{\partial}\right)^{\frac{1}{\partial}}\right\}}, \\ \sqrt{1 - exp\left\{-\left(\sum_{\lambda=1}^{n} \varpi_{\lambda} \left(-ln\left(1 - \left(\delta_{\lambda}^+\right)^2\right)\right)^{\partial}\right)^{\frac{1}{\partial}}\right\}} \end{array} \right], \\ \sqrt{1 - exp\left\{-\left(\sum_{\lambda=1}^{n} \varpi_{\lambda} \left(-ln(1 - (\delta_{\lambda})^2)\right)^{\partial}\right)^{\frac{1}{\partial}}\right\}} \right), \\ \left(\left[\begin{array}{c} \sqrt{exp\left\{-\left(\sum_{\lambda=1}^{n} \varpi_{\lambda} \left(-ln((\varepsilon_{\lambda}^-)^2)\right)^{\partial}\right)^{\frac{1}{\partial}}\right\}}, \\ \sqrt{exp\left\{-\left(\sum_{\lambda=1}^{n} \varpi_{\lambda} \left(-ln\left(\left(\varepsilon_{\lambda}^+\right)^2\right)\right)^{\partial}\right)^{\frac{1}{\partial}}\right\}} \end{array} \right], \\ \sqrt{exp\left\{-\left(\sum_{\lambda=1}^{n} \varpi_{\lambda} \left(-ln((\varepsilon_{\lambda})^2)\right)^{\partial}\right)^{\frac{1}{\partial}}\right\}} \right), \\ \left(\left[\begin{array}{c} \sqrt{exp\left\{-\left(\sum_{\lambda=1}^{n} \varpi_{\lambda} \left(-ln((\phi_{\lambda}^-)^2)\right)^{\partial}\right)^{\frac{1}{\partial}}\right\}}, \\ \sqrt{exp\left\{-\left(\sum_{\lambda=1}^{n} \varpi_{\lambda} \left(-ln\left(\left(\phi_{\lambda}^+\right)^2\right)\right)^{\partial}\right)^{\frac{1}{\partial}}\right\}} \end{array} \right], \\ \sqrt{exp\left\{-\left(\sum_{\lambda=1}^{n} \varpi_{\lambda} \left(-ln((\phi_{\lambda})^2)\right)^{\partial}\right)^{\frac{1}{\partial}}\right\}} \right) \end{array} \right\} \tag{11}$$

Proof. This theorem can be proved with the aid of mathematical induction principle. Obviously, the Eq.(11) holds for $n = 1$. When $n = 2$, one has

$$\varpi_1\Theta_1 = \left\{\begin{array}{c} \left(\begin{array}{c}\left[\begin{array}{c}\sqrt{1-exp\left\{-(\varpi_1(-\,ln(1-(\delta_1^-)^2))^{\partial})^{\frac{1}{\partial}}\right\}},\\ \sqrt{1-exp\left\{-(\varpi_1(-\,ln(1-(\delta_1^+)^2))^{\partial})^{\frac{1}{\partial}}\right\}}\end{array}\right],\\ \sqrt{1-exp\left\{-(\varpi_1(-\,ln(1-(\delta_1)^2))^{\partial})^{\frac{1}{\partial}}\right\}}\end{array}\right),\\ \left(\begin{array}{c}\left[\begin{array}{c}\sqrt{exp\left\{-(\varpi_1(-\,ln((\varepsilon_1^-)^2))^{\partial})^{\frac{1}{\partial}}\right\}},\\ \sqrt{exp\left\{-(\varpi_1(-\,ln((\varepsilon_1^+)^2))^{\partial})^{\frac{1}{\partial}}\right\}}\end{array}\right],\\ \sqrt{1-exp\left\{-(\varpi_1(-\,ln((\varepsilon_1)^2))^{\partial})^{\frac{1}{\partial}}\right\}}\end{array}\right),\\ \left(\begin{array}{c}\left[\begin{array}{c}\sqrt{exp\left\{-(\varpi_1(-\,ln((\phi_1^-)^2))^{\partial})^{\frac{1}{\partial}}\right\}},\\ \sqrt{exp\left\{-(\varpi_1(-\,ln((\phi_1^+)^2))^{\partial})^{\frac{1}{\partial}}\right\}}\end{array}\right],\\ \sqrt{exp\left\{-(\varpi_1(-\,ln((\phi_1)^2))^{\partial})^{\frac{1}{\partial}}\right\}}\end{array}\right)\end{array}\right\},$$

$$\varpi_2\Theta_2 = \left\{\begin{array}{c} \left(\begin{array}{c}\left[\begin{array}{c}\sqrt{1-exp\left\{-(\varpi_2(-\,ln(1-(\delta_2^-)^2))^{\partial})^{\frac{1}{\partial}}\right\}},\\ \sqrt{1-exp\left\{-(\varpi_2(-\,ln(1-(\delta_2^+)^2))^{\partial})^{\frac{1}{\partial}}\right\}}\end{array}\right],\\ \sqrt{1-exp\left\{-(\varpi_2(-\,ln(1-(\delta_2)^2))^{\partial})^{\frac{1}{\partial}}\right\}}\end{array}\right),\\ \left(\begin{array}{c}\left[\begin{array}{c}\sqrt{exp\left\{-(\varpi_2(-\,ln((\varepsilon_2^-)^2))^{\partial})^{\frac{1}{\partial}}\right\}},\\ \sqrt{exp\left\{-(\varpi_2(-\,ln((\varepsilon_2^+)^2))^{\partial})^{\frac{1}{\partial}}\right\}}\end{array}\right],\\ \sqrt{1-exp\left\{-(\varpi_2(-\,ln((\varepsilon_{12})^2))^{\partial})^{\frac{1}{\partial}}\right\}}\end{array}\right),\\ \left(\begin{array}{c}\left[\begin{array}{c}\sqrt{exp\left\{-(\varpi_2(-\,ln((\phi_2^-)^2))^{\partial})^{\frac{1}{\partial}}\right\}},\\ \sqrt{exp\left\{-(\varpi_2(-\,ln((\phi_2^+)^2))^{\partial})^{\frac{1}{\partial}}\right\}}\end{array}\right],\\ \sqrt{exp\left\{-(\varpi_2(-\,ln((\phi_2)^2))^{\partial})^{\frac{1}{\partial}}\right\}}\end{array}\right)\end{array}\right\}.$$

Therefore, we can acquire

$$SCFAAWA(\Theta_1,\Theta_2)=\varpi_1\Theta_1\oplus\varpi_2\Theta_2=\left\{\begin{array}{c}\left(\left[\begin{array}{c}\sqrt{1-exp\left\{-\left(\sum_{\lambda=1}^{2}\varpi_\lambda\left(-ln(1-(\delta_\lambda^-)^2)\right)^\partial\right)^{\frac{1}{\partial}}\right\}},\\ \sqrt{1-exp\left\{-\left(\sum_{\lambda=1}^{2}\varpi_\lambda\left(-ln\left(1-\left(\delta_\lambda^+\right)^2\right)\right)^\partial\right)^{\frac{1}{\partial}}\right\}}\end{array}\right], \sqrt{1-exp\left\{-\left(\sum_{\lambda=1}^{2}\varpi_\lambda\left(-ln(1-(\delta_\lambda)^2)\right)^\partial\right)^{\frac{1}{\partial}}\right\}}\right),\\ \left(\left[\begin{array}{c}\sqrt{exp\left\{-\left(\sum_{\lambda=1}^{2}\varpi_\lambda\left(-ln((\varepsilon_\lambda^-)^2)\right)^\partial\right)^{\frac{1}{\partial}}\right\}},\\ \sqrt{exp\left\{-\left(\sum_{\lambda=1}^{2}\varpi_\lambda\left(-ln\left(\left(\varepsilon_\lambda^+\right)^2\right)\right)^\partial\right)^{\frac{1}{\partial}}\right\}}\end{array}\right], \sqrt{exp\left\{-\left(\sum_{\lambda=1}^{2}\varpi_\lambda\left(-ln((\varepsilon_\lambda)^2)\right)^\partial\right)^{\frac{1}{\partial}}\right\}}\right),\\ \left(\left[\begin{array}{c}\sqrt{exp\left\{-\left(\sum_{\lambda=1}^{2}\varpi_\lambda\left(-ln((\phi_\lambda^-)^2)\right)^\partial\right)^{\frac{1}{\partial}}\right\}},\\ \sqrt{exp\left\{-\left(\sum_{\lambda=1}^{2}\varpi_\lambda\left(-ln\left(\left(\phi_\lambda^+\right)^2\right)\right)^\partial\right)^{\frac{1}{\partial}}\right\}}\end{array}\right], \sqrt{exp\left\{-\left(\sum_{\lambda=1}^{2}\varpi_\lambda\left(-ln((\phi_\lambda)^2)\right)^\partial\right)^{\frac{1}{\partial}}\right\}}\right)\end{array}\right\}$$

Thus, Eq. (11) keeps for $n=2$. It is supposed that Eq. (11) keeps for $n=\hat{n}$, then

$$SCFAAWA(\Theta_1,\Theta_2,\ldots,\Theta_{\hat{n}})=\left\{\begin{array}{l}\left(\left[\begin{array}{l}\sqrt{1-exp\left\{-\left(\sum_{\lambda=1}^{\hat{n}}\varpi_{\lambda}\left(-ln(1-(\delta_{\lambda}^{-})^2)\right)^{\partial}\right)^{\frac{1}{\partial}}\right\}},\\ \sqrt{1-exp\left\{-\left(\sum_{\lambda=1}^{\hat{n}}\varpi_{\lambda}\left(-ln\left(1-\left(\delta_{\lambda}^{+}\right)^2\right)\right)^{\partial}\right)^{\frac{1}{\partial}}\right\}}\end{array}\right],\sqrt{1-exp\left\{-\left(\sum_{\lambda=1}^{\hat{n}}\varpi_{\lambda}\left(-ln(1-(\delta_{\lambda})^2)\right)^{\partial}\right)^{\frac{1}{\partial}}\right\}}\right),\\ \left(\left[\begin{array}{l}\sqrt{exp\left\{-\left(\sum_{\lambda=1}^{\hat{n}}\varpi_{\lambda}\left(-ln((\varepsilon_{\lambda}^{-})^2)\right)^{\partial}\right)^{\frac{1}{\partial}}\right\}},\\ \sqrt{exp\left\{-\left(\sum_{\lambda=1}^{\hat{n}}\varpi_{\lambda}\left(-ln\left(\left(\varepsilon_{\lambda}^{+}\right)^2\right)\right)^{\partial}\right)^{\frac{1}{\partial}}\right\}}\end{array}\right],\sqrt{exp\left\{-\left(\sum_{\lambda=1}^{\hat{n}}\varpi_{\lambda}\left(-ln((\varepsilon_{\lambda})^2)\right)^{\partial}\right)^{\frac{1}{\partial}}\right\}}\right),\\ \left(\left[\begin{array}{l}\sqrt{exp\left\{-\left(\sum_{\lambda=1}^{\hat{n}}\varpi_{\lambda}\left(-ln((\phi_{\lambda}^{-})^2)\right)^{\partial}\right)^{\frac{1}{\partial}}\right\}},\\ \sqrt{exp\left\{-\left(\sum_{\lambda=1}^{\hat{n}}\varpi_{\lambda}\left(-ln\left(\left(\phi_{\lambda}^{+}\right)^2\right)\right)^{\partial}\right)^{\frac{1}{\partial}}\right\}}\end{array}\right],\sqrt{exp\left\{-\left(\sum_{\lambda=1}^{\hat{n}}\varpi_{\lambda}\left(-ln((\phi_{\lambda})^2)\right)^{\partial}\right)^{\frac{1}{\partial}}\right\}}\right)\end{array}\right\}.$$

At present, for $n=\hat{n}+1$, we have

$$SCFAAWA(\Theta_1,\Theta_2,\ldots,\Theta_{\hat{n}+1})=SCFAAWA(\Theta_1,\Theta_2,\ldots,\Theta_{\hat{n}})\oplus\varpi_{\hat{n}+1}\Theta_{\hat{n}+1}$$

$$=\left\{\begin{array}{l}\left(\left[\begin{array}{l}\sqrt{1-exp\left\{-\left(\sum_{\lambda=1}^{\hat{n}+1}\varpi_{\lambda}\left(-ln(1-(\delta_{\lambda}^{-})^{2})\right)^{\partial}\right)^{\frac{1}{\partial}}\right\}},\\ \sqrt{1-exp\left\{-\left(\sum_{\lambda=1}^{\hat{n}+1}\varpi_{\lambda}\left(-ln\left(1-\left(\delta_{\lambda}^{+}\right)^{2}\right)\right)^{\partial}\right)^{\frac{1}{\partial}}\right\}}\end{array}\right],\sqrt{1-exp\left\{-\left(\sum_{\lambda=1}^{\hat{n}+1}\varpi_{\lambda}\left(-ln(1-(\delta_{\lambda})^{2})\right)^{\partial}\right)^{\frac{1}{\partial}}\right\}}\right),\\ \left(\left[\begin{array}{l}\sqrt{exp\left\{-\left(\sum_{\lambda=1}^{\hat{n}+1}\varpi_{\lambda}\left(-ln((\varepsilon_{\lambda}^{-})^{2})\right)^{\partial}\right)^{\frac{1}{\partial}}\right\}},\\ \sqrt{exp\left\{-\left(\sum_{\lambda=1}^{\hat{n}+1}\varpi_{\lambda}\left(-ln\left(\left(\varepsilon_{\lambda}^{+}\right)^{2}\right)\right)^{\partial}\right)^{\frac{1}{\partial}}\right\}}\end{array}\right],\sqrt{exp\left\{-\left(\sum_{\lambda=1}^{\hat{n}+1}\varpi_{\lambda}\left(-ln((\varepsilon_{\lambda})^{2})\right)^{\partial}\right)^{\frac{1}{\partial}}\right\}}\right),\\ \left(\left[\begin{array}{l}\sqrt{exp\left\{-\left(\sum_{\lambda=1}^{\hat{n}+1}\varpi_{\lambda}\left(-ln((\phi_{\lambda}^{-})^{2})\right)^{\partial}\right)^{\frac{1}{\partial}}\right\}},\\ \sqrt{exp\left\{-\left(\sum_{\lambda=1}^{\hat{n}+1}\varpi_{\lambda}\left(-ln\left(\left(\phi_{\lambda}^{+}\right)^{2}\right)\right)^{\partial}\right)^{\frac{1}{\partial}}\right\}}\end{array}\right],\sqrt{exp\left\{-\left(\sum_{\lambda=1}^{\hat{n}+1}\varpi_{\lambda}\left(-ln((\phi_{\lambda})^{2})\right)^{\partial}\right)^{\frac{1}{\partial}}\right\}}\right)\end{array}\right\}.$$

Hence, Eq. (11) keeps for $n=\hat{n}+1$. Thus, Eq.(11) holds for all n.

Definition 13. Let $\Theta_{\lambda}=\{([\delta_{\lambda}^{-},\delta_{\lambda}^{+}],\delta_{\lambda}),([\varepsilon_{\lambda}^{-},\varepsilon_{\lambda}^{+}],\varepsilon_{\lambda}),([\phi_{\lambda}^{-},\phi_{\lambda}^{+}],\phi_{\lambda})\}(\lambda=1,2,\cdots,n)$ be a set of SCFNs. ϖ_{λ} is the importance of SCFN Θ_{λ} with $\sum_{\lambda=1}^{n}\varpi_{\lambda}=1,\varpi_{\lambda}\in[0,1]$. Then SCF Aczel–Alsina ordered weighted averaging (SCFAAOWA) operator is depicted as:

$$SCFAAOWA(\Theta_1,\Theta_2,\ldots,\Theta_n)=\varpi_1\Theta_{o(1)}\oplus\varpi_2\Theta_{o(2)}\oplus\cdots\oplus\varpi_n\Theta_{o(n)}, \tag{12}$$

where $\left(o(1), o(2), \cdots, o(n)\right)$ stands for the permutation of $(ƛ = 1,2,\cdots,n)$ with $\Theta_{o(ƛ-1)} \geq \Theta_{o(ƛ)}$, $\forall ƛ = 2,3,\cdots,n$.

Theorem 3. Let $\Theta_ƛ = \{([\delta_ƛ^-, \delta_ƛ^+], \delta_ƛ), ([\varepsilon_ƛ^-, \varepsilon_ƛ^+], \varepsilon_ƛ), ([\phi_ƛ^-, \phi_ƛ^+], \phi_ƛ)\}(ƛ = 1,2,\cdots,n)$ be a set of SCFNs. Then the consolidated value of the set of SCFNs by SCFAAOWA operator is a SCFNs and portrayed as:

$$SCFAAOWA(\Theta_1, \Theta_2, \ldots, \Theta_n) =$$

$$\left\{\begin{array}{l}
\left(\left[\begin{array}{l}\sqrt{1-exp\left\{-\left(\sum_{ƛ=1}^{n}\varpi_ƛ\left(-ln\left(1-\left(\delta_{o(ƛ)}^-\right)^2\right)\right)^{\partial}\right)^{\frac{1}{\partial}}\right\}},\\ \sqrt{1-exp\left\{-\left(\sum_{ƛ=1}^{n}\varpi_ƛ\left(-ln\left(1-\left(\delta_{o(ƛ)}^+\right)^2\right)\right)^{\partial}\right)^{\frac{1}{\partial}}\right\}}\end{array}\right], \sqrt{1-exp\left\{-\left(\sum_{ƛ=1}^{n}\varpi_ƛ\left(-ln\left(1-\left(\delta_{o(ƛ)}\right)^2\right)\right)^{\partial}\right)^{\frac{1}{\partial}}\right\}}\right),\\
\left(\left[\begin{array}{l}\sqrt{exp\left\{-\left(\sum_{ƛ=1}^{n}\varpi_ƛ\left(-ln\left(\left(\varepsilon_{o(ƛ)}^-\right)^2\right)\right)^{\partial}\right)^{\frac{1}{\partial}}\right\}},\\ \sqrt{exp\left\{-\left(\sum_{ƛ=1}^{n}\varpi_ƛ\left(-ln\left(\left(\varepsilon_{o(ƛ)}^+\right)^2\right)\right)^{\partial}\right)^{\frac{1}{\partial}}\right\}}\end{array}\right], \sqrt{exp\left\{-\left(\sum_{ƛ=1}^{n}\varpi_ƛ\left(-ln\left(\left(\varepsilon_{o(ƛ)}\right)^2\right)\right)^{\partial}\right)^{\frac{1}{\partial}}\right\}}\right),\\
\left(\left[\begin{array}{l}\sqrt{exp\left\{-\left(\sum_{ƛ=1}^{n}\varpi_ƛ\left(-ln\left(\left(\phi_{o(ƛ)}^-\right)^2\right)\right)^{\partial}\right)^{\frac{1}{\partial}}\right\}},\\ \sqrt{exp\left\{-\left(\sum_{ƛ=1}^{n}\varpi_ƛ\left(-ln\left(\left(\phi_{o(ƛ)}^+\right)^2\right)\right)^{\partial}\right)^{\frac{1}{\partial}}\right\}}\end{array}\right], \sqrt{exp\left\{-\left(\sum_{ƛ=1}^{n}\varpi_ƛ\left(-ln\left(\left(\phi_{o(ƛ)}\right)^2\right)\right)^{\partial}\right)^{\frac{1}{\partial}}\right\}}\right)
\end{array}\right\} \tag{13}$$

Proposition 1. Let $\Theta_ƛ = \{([\delta_ƛ^-, \delta_ƛ^+], \delta_ƛ), ([\varepsilon_ƛ^-, \varepsilon_ƛ^+], \varepsilon_ƛ), ([\phi_ƛ^-, \phi_ƛ^+], \phi_ƛ)\}(ƛ = 1,2,\cdots,n)$ be a set of SCFNs. $\varpi_ƛ$ is the importance of SCFN $\Theta_ƛ$ with $\sum_{ƛ=1}^{n}\varpi_ƛ = 1$, $\varpi_ƛ \in [0,1]$. Then SCFAAWA and SCFAAOWA operators meet the following properties:

(1) Idempotency: If all SCFNs are equal, i.e., $\Theta_ƛ = \Theta$, $\forall ƛ$, then one has

$$SCFAAWA(\Theta_1, \Theta_2, \ldots, \Theta_n) = \Theta,$$
$$SCFAAOWA(\Theta_1, \Theta_2, \ldots, \Theta_n) = \Theta.$$

(2) Boundedness: if $\Theta^- = min\{\Theta_1, \Theta_2, \cdots, \Theta_n\}$ and $\Theta^+ = max\{\Theta_1, \Theta_2, \cdots, \Theta_n\}$, then we have

$$\Theta^- \leq SCFAAWA(\Theta_1, \Theta_2, \ldots, \Theta_n) \leq \Theta^+,$$
$$\Theta^- \leq SCFAAOWA(\Theta_1, \Theta_2, \ldots, \Theta_n) \leq \Theta^+.$$

(3) Monotonicity: For the set of SCFNs, $\Theta_ƛ$ and $\widehat{\Theta}_ƛ$ $(ƛ = 1,2,\cdots,n)$. If $\Theta_ƛ \leq \widehat{\Theta}_ƛ$, then we have

$$SCFAAWA(\Theta_1, \Theta_2, \ldots, \Theta_n) \leq SCFAAWA\left(\widehat{\Theta}_1, \widehat{\Theta}_2, \ldots, \widehat{\Theta}_n\right),$$
$$SCFAAOWA(\Theta_1, \Theta_2, \ldots, \Theta_n) \leq SCFAAOWA\left(\widehat{\Theta}_1, \widehat{\Theta}_2, \ldots, \widehat{\Theta}_n\right).$$

3.2 SCF Aczel–Alsina weighted geometric operator

The current section develops SCF Aczel–Alsina weighted and ordered weighted geometric operators utilizing the Aczel–Alsina norms.

Definition 14. Let $\Theta_ƛ = \{([\delta_ƛ^-, \delta_ƛ^+], \delta_ƛ), ([\varepsilon_ƛ^-, \varepsilon_ƛ^+], \varepsilon_ƛ), ([\phi_ƛ^-, \phi_ƛ^+], \phi_ƛ)\}(ƛ = 1,2,\cdots,n)$ be a set of SCFNs. where $\varpi_ƛ$ is the importance of SCFN $\Theta_ƛ$ with $\sum_{ƛ=1}^{n} \varpi_ƛ = 1, \varpi_ƛ \in [0,1]$. Then SCF Aczel–Alsina weighted geometric (SCFAAWG) operator is depicted as:

$$SCFAAWG(\Theta_1, \Theta_2, \ldots, \Theta_n) = (\Theta_1)^{\varpi_1} \otimes (\Theta_2)^{\varpi_2} \otimes \cdots \otimes (\Theta_n)^{\varpi_n}. \quad (14)$$

Theorem 4. Let $\Theta_ƛ = \{([\delta_ƛ^-, \delta_ƛ^+], \delta_ƛ), ([\varepsilon_ƛ^-, \varepsilon_ƛ^+], \varepsilon_ƛ), ([\phi_ƛ^-, \phi_ƛ^+], \phi_ƛ)\}(ƛ = 1,2,\cdots,n)$ be a set of SCFNs. Then the consolidated outcome of them by SCFAAWG operator is a SCFN and represented as:

$$SCFAAWG(\Theta_1, \Theta_2, \ldots, \Theta_n) = \left\{ \begin{array}{c} \left(\left[\begin{array}{l} \sqrt{exp\left\{-\left(\sum_{ƛ=1}^{n} \varpi_ƛ \left(-ln((\delta_ƛ^-)^2)\right)^{\partial}\right)^{\frac{1}{\partial}}\right\}}, \\ \sqrt{exp\left\{-\left(\sum_{ƛ=1}^{n} \varpi_ƛ \left(-ln\left(\left(\delta_ƛ^+\right)^2\right)\right)^{\partial}\right)^{\frac{1}{\partial}}\right\}} \end{array} \right], \sqrt{exp\left\{-\left(\sum_{ƛ=1}^{n} \varpi_ƛ \left(-ln((\delta_ƛ)^2)\right)^{\partial}\right)^{\frac{1}{\partial}}\right\}} \right), \\ \left(\left[\begin{array}{l} \sqrt{exp\left\{-\left(\sum_{ƛ=1}^{n} \varpi_ƛ \left(-ln((\varepsilon_ƛ^-)^2)\right)^{\partial}\right)^{\frac{1}{\partial}}\right\}}, \\ \sqrt{exp\left\{-\left(\sum_{ƛ=1}^{n} \varpi_ƛ \left(-ln\left(\left(\varepsilon_ƛ^+\right)^2\right)\right)^{\partial}\right)^{\frac{1}{\partial}}\right\}} \end{array} \right], \sqrt{exp\left\{-\left(\sum_{ƛ=1}^{n} \varpi_ƛ \left(-ln((\varepsilon_ƛ)^2)\right)^{\partial}\right)^{\frac{1}{\partial}}\right\}} \right), \\ \left(\left[\begin{array}{l} \sqrt{1 - exp\left\{-\left(\sum_{ƛ=1}^{n} \varpi_ƛ \left(-ln(1-(\phi_ƛ^-)^2)\right)^{\partial}\right)^{\frac{1}{\partial}}\right\}}, \\ \sqrt{1 - exp\left\{-\left(\sum_{ƛ=1}^{n} \varpi_ƛ \left(-ln\left(1-\left(\phi_ƛ^+\right)^2\right)\right)^{\partial}\right)^{\frac{1}{\partial}}\right\}} \end{array} \right], \sqrt{1 - exp\left\{-\left(\sum_{ƛ=1}^{n} \varpi_ƛ \left(-ln(1-(\phi_ƛ)^2)\right)^{\partial}\right)^{\frac{1}{\partial}}\right\}} \right) \end{array} \right\}. \quad (15)$$

Proof. It is similar to Theorem 2.

Definition 15. Let $\Theta_ƛ = \{([\delta_ƛ^-, \delta_ƛ^+], \delta_ƛ), ([\varepsilon_ƛ^-, \varepsilon_ƛ^+], \varepsilon_ƛ), ([\phi_ƛ^-, \phi_ƛ^+], \phi_ƛ)\}(ƛ = 1,2,\cdots,n)$ be a set of SCFNs. $\varpi_ƛ$ is the importance of SCFN $\Theta_ƛ$ with $\sum_{ƛ=1}^{n} \varpi_ƛ = 1, \varpi_ƛ \in [0,1]$. Then SCF Aczel–Alsina ordered weighted geometric (SCFAAOWG) operator is depicted as:

$$SCFAAOWG(\Theta_1, \Theta_2, \ldots, \Theta_n) = \varpi_1 \Theta_{o(1)} \oplus \varpi_2 \Theta_{o(2)} \oplus \cdots \oplus \varpi_n \Theta_{o(n)}, \quad (16)$$

where $(o(1), o(2), \cdots, o(n))$ stands for the permutation of $(ƛ = 1,2,\cdots,n)$ with $\Theta_{o(ƛ-1)} \geq \Theta_{o(ƛ)}$, $\forall ƛ = 2,3,\cdots,n$.

Theorem 5. Let $\Theta_ƛ = \{([\delta_ƛ^-, \delta_ƛ^+], \delta_ƛ), ([\varepsilon_ƛ^-, \varepsilon_ƛ^+], \varepsilon_ƛ), ([\phi_ƛ^-, \phi_ƛ^+], \phi_ƛ)\}(ƛ = 1,2,\cdots,n)$ be a set of SCFNs. Then the consolidated value of the set of SCFNs by SCFAAOWG operator is a SCFNs and portrayed as:

$$SCFAAOWG(\Theta_1,\Theta_2,\dots,\Theta_n)=$$
$$\left\{\begin{array}{l}\left(\begin{array}{l}\left[\begin{array}{l}\sqrt{exp\left\{-\left(\sum_{\lambda=1}^{n}\varpi_\lambda\left(-ln\left(\left(\delta_{o(\lambda)}^{-}\right)^2\right)\right)^{\partial}\right)^{\frac{1}{\partial}}\right\}},\\ \sqrt{exp\left\{-\left(\sum_{\lambda=1}^{n}\varpi_\lambda\left(-ln\left(\left(\delta_{o(\lambda)}^{+}\right)^2\right)\right)^{\partial}\right)^{\frac{1}{\partial}}\right\}}\end{array}\right],\\ \sqrt{exp\left\{-\left(\sum_{\lambda=1}^{n}\varpi_\lambda\left(-ln\left(\left(\delta_{o(\lambda)}\right)^2\right)\right)^{\partial}\right)^{\frac{1}{\partial}}\right\}}\end{array}\right)\\ \left(\begin{array}{l}\left[\begin{array}{l}\sqrt{exp\left\{-\left(\sum_{\lambda=1}^{n}\varpi_\lambda\left(-ln\left(\left(\varepsilon_{o(\lambda)}^{-}\right)^2\right)\right)^{\partial}\right)^{\frac{1}{\partial}}\right\}},\\ \sqrt{exp\left\{-\left(\sum_{\lambda=1}^{n}\varpi_\lambda\left(-ln\left(\left(\varepsilon_{o(\lambda)}^{+}\right)^2\right)\right)^{\partial}\right)^{\frac{1}{\partial}}\right\}}\end{array}\right],\\ \sqrt{exp\left\{-\left(\sum_{\lambda=1}^{n}\varpi_\lambda\left(-ln\left(\left(\varepsilon_{o(\lambda)}\right)^2\right)\right)^{\partial}\right)^{\frac{1}{\partial}}\right\}}\end{array}\right),\\ \left(\begin{array}{l}\left[\begin{array}{l}\sqrt{1-exp\left\{-\left(\sum_{\lambda=1}^{n}\varpi_\lambda\left(-ln\left(1-\left(\phi_{o(\lambda)}^{-}\right)^2\right)\right)^{\partial}\right)^{\frac{1}{\partial}}\right\}},\\ \sqrt{1-exp\left\{-\left(\sum_{\lambda=1}^{n}\varpi_\lambda\left(-ln\left(1-\left(\phi_{o(\lambda)}^{+}\right)^2\right)\right)^{\partial}\right)^{\frac{1}{\partial}}\right\}}\end{array}\right],\\ \sqrt{1-exp\left\{-\left(\sum_{\lambda=1}^{n}\varpi_\lambda\left(-ln\left(1-\left(\phi_{o(\lambda)}\right)^2\right)\right)^{\partial}\right)^{\frac{1}{\partial}}\right\}}\end{array}\right)\end{array}\right\}. \tag{17}$$

Proof. It is similar to Theorem 2.

Proposition 2. Let $\Theta_\lambda=\{([\delta_\lambda^-,\delta_\lambda^+],\delta_\lambda),([\varepsilon_\lambda^-,\varepsilon_\lambda^+],\varepsilon_\lambda),([\phi_\lambda^-,\phi_\lambda^+],\phi_\lambda)\}(\lambda=1,2,\cdots,n)$ be a set of SCFNs. ϖ_λ is the importance of SCFN Θ_λ with $\sum_{\lambda=1}^{n}\varpi_\lambda=1,\varpi_\lambda\in[0,1]$. Then SCFAAWG and SCFAAOWG operators meet the following properties:

(1) Idempotency: If all SCFNs are equal, i.e., $\Theta_\lambda=\Theta$, $\forall\lambda$, then one has

$$SCFAAWG(\Theta_1,\Theta_2,\dots,\Theta_n)=\Theta,$$
$$SCFAAOWG(\Theta_1,\Theta_2,\dots,\Theta_n)=\Theta.$$

(2) Boundedness: if $\Theta^-=min\{\Theta_1,\Theta_2,\cdots,\Theta_n\}$ and $\Theta^+=max\{\Theta_1,\Theta_2,\cdots,\Theta_n\}$, then we have

$$\Theta^-\le SCFAAWG(\Theta_1,\Theta_2,\dots,\Theta_n)\le\Theta^+,$$
$$\Theta^-\le SCFAAOWG(\Theta_1,\Theta_2,\dots,\Theta_n)\le\Theta^+.$$

(3) Monotonicity: For the set of SCFNs, Θ_λ and $\widehat{\Theta}_\lambda$ $(\lambda=1,2,\cdots,n)$. If $\Theta_\lambda\le\widehat{\Theta}_\lambda$, then we have

$$SCFAAWG(\Theta_1,\Theta_2,\dots,\Theta_n)\le SCFAAWG\left(\widehat{\Theta}_1,\widehat{\Theta}_2,\dots,\widehat{\Theta}_n\right),$$
$$SCFAAOWG(\Theta_1,\Theta_2,\dots,\Theta_n)\le SCFAAOWG\left(\widehat{\Theta}_1,\widehat{\Theta}_2,\dots,\widehat{\Theta}_n\right).$$

4. An enhanced decision approach under SCF setting

In this section, we propose a novel SCF decision approach based on Reni entropy method and improved SCF-WASPAS method. In the proposed MCDM method, the weight information is determined by used the SCF-Renyi entropy method. The ranking of alternatives can be ascertained by the SCFAAWA and SCFAAWG operators method-based WASPAS method.

To deal with the MCDM problem with SCF information, several important notions of classical MCDM problem are given in advance. For a traditional MCDM problem, it is made up of a set of schemes denoted as $G = \{G_i | i = 1,2,\cdots,m\}$ and a set of criteria denoted as $C = \{C_ƛ | ƛ = 1,2,\cdots,n\}$. It is assumed that decision expert utilizes the SCF information to express their preference for schemes with respect to assessment criteria, then the SCF decision matrix is constructed as $Q = (q_{iƛ})_{m\times n}$, where $q_{iƛ} = \{([\delta_{iƛ}^-, \delta_{iƛ}^+], \delta_{iƛ}), ([\varepsilon_{iƛ}^-, \varepsilon_{iƛ}^+], \varepsilon_{iƛ}), ([\phi_{iƛ}^-, \phi_{iƛ}^+], \phi_{iƛ})\}$ is a SCFN and represents the assessment of decision alternative$G_i$with respect to attribute$C_ƛ$. Based on the mentioned notions, the steps of the proposed SCF-Renyi-WASPAS method can be illustrated as below.

Step 1： Attaining the SCF decision-making matrix $Q = (q_{iƛ})_{m\times n} (i = 1,2,\cdots m; ƛ = 1,2,\cdots n)$.

$$H = (q_{iƛ})_{m\times n} = \begin{pmatrix} \{([\delta_{11}^-, \delta_{11}^+], \delta_{11}), ([\varepsilon_{11}^-, \varepsilon_{11}^+], \varepsilon_{11}), ([\phi_{11}^-, \phi_{11}^+], \phi_{11})\} & \cdots & \{([\delta_{1n}^-, \delta_{1n}^+], \delta_{1n}), ([\varepsilon_{1n}^-, \varepsilon_{1n}^+], \varepsilon_{1n}), ([\phi_{1n}^-, \phi_{1n}^+], \phi_{1n})\} \\ \{([\delta_{21}^-, \delta_{21}^+], \delta_{21}), ([\varepsilon_{21}^-, \varepsilon_{21}^+], \varepsilon_{21}), ([\phi_{21}^-, \phi_{21}^+], \phi_{21})\} & \cdots & \{([\delta_{2n}^-, \delta_{2n}^+], \delta_{2n}), ([\varepsilon_{2n}^-, \varepsilon_{2n}^+], \varepsilon_{2n}), ([\phi_{2n}^-, \phi_{2n}^+], \phi_{2n})\} \\ \vdots & \ddots & \vdots \\ \{([\delta_{m1}^-, \delta_{m1}^+], \delta_{m1}), ([\varepsilon_{m1}^-, \varepsilon_{m1}^+], \varepsilon_{m1}), ([\phi_{m1}^-, \phi_{m1}^+], \phi_{m1})\} & \cdots & \{([\delta_{mn}^-, \delta_{mn}^+], \delta_{mn}), ([\varepsilon_{mn}^-, \varepsilon_{mn}^+], \varepsilon_{mn}), ([\phi_{mn}^-, \phi_{mn}^+], \phi_{mn})\} \end{pmatrix}. \quad (18)$$

Step 2: Because the criteria in MCDM problem has two types including benefit-type and cost-type criteria. Hence, we need to shift the cost-type criteria to benefit-type criteria and then attain the normalized SCF decision matrix. Therefore, the SCF decision matrix $Q = (q_{iƛ})_{m\times n} (i = 1,2,\cdots m; ƛ = 1,2,\cdots n)$ is shifted into normalized SCF decision matrix (NSCFDM) $\tilde{Q} = (\tilde{q}_{iƛ})_{m\times n} (i = 1,2,\cdots m; ƛ = 1,2,\cdots n)$ by the following formulation:

$$\tilde{q}_{iƛ} = \begin{cases} q_{iƛ} = \{([\delta_{iƛ}^-, \delta_{iƛ}^+], \delta_{iƛ}), ([\varepsilon_{iƛ}^-, \varepsilon_{iƛ}^+], \varepsilon_{iƛ}), ([\phi_{iƛ}^-, \phi_{iƛ}^+], \phi_{iƛ})\}, \text{for benefit criteria} \\ (q_{iƛ})^c = \{([\phi_{iƛ}^-, \phi_{iƛ}^+], \phi_{iƛ}),, ([\varepsilon_{iƛ}^-, \varepsilon_{iƛ}^+], \varepsilon_{iƛ}), ([\delta_{iƛ}^-, \delta_{iƛ}^+], \delta_{iƛ})\}, \text{for cost criteria} \end{cases}. \quad (19)$$

Step 3: Compute the weight of criteria.

Step 3.1: Figure out the score matrix $SF = (sf_{iƛ})_{m\times n}$of the NSCFDM by Eq.(20):

$$sf_{iƛ} = \frac{1}{2}\left(1 + \frac{1}{9}((\delta_{iƛ}^- + \delta_{iƛ}^+ + \delta_{iƛ})^2 + (\varepsilon_{iƛ}^- + \varepsilon_{iƛ}^+ + \varepsilon_{iƛ})^2 - (\phi_{iƛ}^- + \phi_{iƛ}^+ + \phi_{iƛ})^2)\right). \quad (20)$$

Step 3.2: Work out the Renyi entropy$E_ƛ$of score function matrix $SF = (sf_{iƛ})_{m\times n}$ by Eq.(21)

$$E_ƛ = \frac{1}{1-\beta} ln\left(\sum_{i=1}^{m} sf_{iƛ}^{\beta}\right). \quad (21)$$

Step 3.3: Compute the objective weight $\varpi_ƛ$ by Eq.(22)

$$\varpi_ƛ = \frac{1-E_ƛ}{\sum_{j=1}^{n}(1-E_ƛ)}. \quad (22)$$

Step 4: Compute the weighted sum measure (WSM) based on SCFAAWA operator, displayed as:

$$\rho_i = SCFAAWA(\tilde{q}_{1\lambda}, \tilde{q}_{2\lambda}, \dots, \tilde{q}_{m\lambda}) =$$

$$\left\{ \begin{array}{l} \left(\left[\begin{array}{l} \sqrt{1 - exp\left\{ -\left(\sum_{i=1}^{m} \varpi_\lambda \left(-ln\left(1 - \left(\delta_{i\lambda}^{-}\right)^2\right)\right)^\partial \right)^{\frac{1}{\partial}} \right\}}, \\ \sqrt{1 - exp\left\{ -\left(\sum_{i=1}^{m} \varpi_\lambda \left(-ln\left(1 - \left(\delta_{i\lambda}^{+}\right)^2\right)\right)^\partial \right)^{\frac{1}{\partial}} \right\}} \end{array} \right], \\ \sqrt{1 - exp\left\{ -\left(\sum_{i=1}^{m} \varpi_\lambda \left(-ln(1 - (\delta_{i\lambda})^2)\right)^\partial \right)^{\frac{1}{\partial}} \right\}} \end{array} \right), \\ \left(\begin{array}{l} \left[\begin{array}{l} \sqrt{exp\left\{ -\left(\sum_{i=1}^{m} \varpi_\lambda \left(-ln\left(\left(\varepsilon_{i\lambda}^{-}\right)^2\right)\right)^\partial \right)^{\frac{1}{\partial}} \right\}}, \\ \sqrt{exp\left\{ -\left(\sum_{i=1}^{m} \varpi_\lambda \left(-ln\left(\left(\varepsilon_{i\lambda}^{+}\right)^2\right)\right)^\partial \right)^{\frac{1}{\partial}} \right\}} \end{array} \right], \\ \sqrt{exp\left\{ -\left(\sum_{i=1}^{m} \varpi_\lambda \left(-ln((\varepsilon_{i\lambda})^2)\right)^\partial \right)^{\frac{1}{\partial}} \right\}} \end{array} \right), \\ \left(\begin{array}{l} \left[\begin{array}{l} \sqrt{exp\left\{ -\left(\sum_{i=1}^{m} \varpi_\lambda \left(-ln\left(\left(\phi_{i\lambda}^{-}\right)^2\right)\right)^\partial \right)^{\frac{1}{\partial}} \right\}}, \\ \sqrt{exp\left\{ -\left(\sum_{i=1}^{m} \varpi_\lambda \left(-ln\left(\left(\phi_{i\lambda}^{+}\right)^2\right)\right)^\partial \right)^{\frac{1}{\partial}} \right\}} \end{array} \right], \\ \sqrt{exp\left\{ -\left(\sum_{i=1}^{n} \varpi_\lambda \left(-ln((\phi_{i\lambda})^2)\right)^\partial \right)^{\frac{1}{\partial}} \right\}} \end{array} \right) \end{array} \right\} \quad (23)$$

Step 5: Compute the weighted product measure (WPM) based on SCFAAWG operator, displayed as

$$\sigma_i = SCFAAWG(\tilde{q}_{1\lambda}, \tilde{q}_{2\lambda}, \dots, \tilde{q}_{m\lambda}) = \left\{ \begin{array}{l} \left(\left[\begin{array}{l} \sqrt{exp\left\{ -\left(\sum_{i=1}^{m} \varpi_\lambda \left(-ln\left((\delta_{i\lambda}^{-})^2 \right) \right)^{\partial} \right)^{\frac{1}{\partial}} \right\}}, \\ \sqrt{exp\left\{ -\left(\sum_{i=1}^{m} \varpi_\lambda \left(-ln\left((\delta_{i\lambda}^{+})^2 \right) \right)^{\partial} \right)^{\frac{1}{\partial}} \right\}} \end{array} \right], \\ \sqrt{exp\left\{ -\left(\sum_{i=1}^{m} \varpi_\lambda \left(-ln((\delta_{i\lambda})^2) \right)^{\partial} \right)^{\frac{1}{\partial}} \right\}} \right), \\ \left(\left[\begin{array}{l} \sqrt{exp\left\{ -\left(\sum_{i=1}^{m} \varpi_{i\lambda} \left(-ln\left((\varepsilon_{i\lambda}^{-})^2 \right) \right)^{\partial} \right)^{\frac{1}{\partial}} \right\}}, \\ \sqrt{exp\left\{ -\left(\sum_{i=1}^{m} \varpi_\lambda \left(-ln\left((\varepsilon_{i\lambda}^{+})^2 \right) \right)^{\partial} \right)^{\frac{1}{\partial}} \right\}} \end{array} \right], \\ \sqrt{exp\left\{ -\left(\sum_{i=1}^{m} \varpi_\lambda \left(-ln((\varepsilon_{i\lambda})^2) \right)^{\partial} \right)^{\frac{1}{\partial}} \right\}} \right), \\ \left(\left[\begin{array}{l} \sqrt{1 - exp\left\{ -\left(\sum_{i=1}^{m} \varpi_\lambda \left(-ln\left(1 - (\phi_{i\lambda}^{-})^2 \right) \right)^{\partial} \right)^{\frac{1}{\partial}} \right\}}, \\ \sqrt{1 - exp\left\{ -\left(\sum_{i=1}^{m} \varpi_\lambda \left(-ln\left(1 - (\phi_{\lambda}^{+})^2 \right) \right)^{\partial} \right)^{\frac{1}{\partial}} \right\}} \end{array} \right], \\ \sqrt{1 - exp\left\{ -\left(\sum_{i=1}^{m} \varpi_\lambda \left(-ln(1 - (\phi_{i\lambda})^2) \right)^{\partial} \right)^{\frac{1}{\partial}} \right\}} \right) \end{array} \right\}. \quad (24)$$

Step 6: Compute the score values of WSM φ_i and WPM $\phi_i$by the following formulation:

$$\varphi_i = \tilde{S}(\rho_i) \quad (25)$$

$$\phi_i = \tilde{S}(\sigma_i) \quad (26)$$

Step 7: Determine the comprehensive assessment value of each scheme.

$$\Xi_i = \chi\varphi_i + (1-\chi)\phi_i, \chi \in [0,1], \quad (27)$$

whereχ is the risk index, when$\chi = 1$, WASPAS reduce to WSM model, when$\chi = 0$, WASPAS reduce to WPM model.

5. Case study

In this section, we present a case study regarding to the assessment of service quality of crowdsourcing logistics platform to show the applicability of the proposed SCF-Renyi-WASPAS method.

5.1 Decision analysis process

A CLP is a logistics service system based on the sharing economy model. It utilizes internet technologies to integrate decentralized social transportation resources (e.g., individuals, part-time delivery personnel, and private vehicles) and completes last-mile delivery, intra-city instant delivery, or specific logistics demands through crowdsourcing. Its core characteristics include: decentralized transportation capacity, dynamic task matching, and flexible participation mechanisms. CLPs have gained significant popularity and substantially enhanced consumer awareness. With the rapid development of the economy, a number of emerging CLPs have emerged. However, due to the lack of standardized policies and regulations, further research is required to determine how to select the optimal crowdsourced logistics service platform. This study examines four of the most prominent crowdsourced logistics service platforms and employs the proposed decision-making method under four evaluation criteria (reliability, empathy, tangibility, and security) to determine their prioritization. The specific steps of the decision-making implementation are as follows:

Step 1： Attaining the SCF decision-making matrix $Q = (q_{i\lambda})_{m\times n}(i = 1,2,\cdots m; \lambda = 1,2,\cdots n)$. Considering the cognition uncertainty of experts, a mapping from linguistic term to SCFNs is provided for expert to express their uncertain assessment, which is displayed in Table 1. Then the assessment matrix can be attained and listed in Table 2. Further, the spherical fuzzy cubic decision matrix is attained and displayed in Table 3.

Table 1

Linguistic terms and their corresponding SCFNs [25]

Extremely high importance	EHI	<([0.8,0.9],0.85), ([0.0,0.1],0.05), ([0.0,0.1],0.05)>
Very high importance	VHI	< ([0.7,0.8],0.75), ([0.1,0.2],0.15), ([0.1,0.2],0.15) >
high importance	HI	< ([0.6,0.7],0.65), ([0.2,0.3],0.25), ([0.2,0.3],0.25) >
Slightly high importance	SHI	< ([0.5,0.6],0.55), ([0.3,0.4],0.35), ([0.3,0.4],0.35) >
Medium importance	MI	< ([0.4,0.5],0.45), ([0.4,0.5],0.45), ([0.4,0.5],0.45) >
Slightly low importance	SLI	< ([0.3,0.4],0.35), ([0.3,0.4],0.35), ([0.5,0.6],0.55) >
low importance	LI	< ([0.2,0.3],0.25), ([0.2,0.3],0.25), ([0.2,0.7],0.65) >
Very low importance	VLI	< ([0.1,0.2],0.15), ([0.1,0.2],0.15), ([0.1,0.8],0.75) >
Extremely low importance	ELI	< ([0.0,0.1],0.05), ([0.0,0.1],0.05), ([0.0,0.9],0.85) >

Table 2

The linguistic assessment for CLPs provided by experts

	C_1	C_2	C_3	C_4
G_1	VHI	HI	MI	HI
G_2	HI	VHI	HI	SLI
G_3	SLI	HI	MI	HI
G_4	MI	HI	VHI	HI

Table 3

The SCF assessment for CLPs provided by experts

	C_1	C_2	C_3	C_4
G_1	<([0.7,0.8],0.75), ([0.1,0.2],0.15), ([0.1,0.2],0.15) >	<([0.6,0.7],0.65), ([0.2,0.3],0.25), ([0.2,0.3],0.25) >	<([0.4,0.5],0.45), ([0.4,0.5],0.45), ([0.4,0.5],0.45) >	<([0.6,0.7],0.65), ([0.2,0.3],0.25), ([0.2,0.3],0.25) >
G_2	<([0.6,0.7],0.65), ([0.2,0.3],0.25), ([0.2,0.3],0.25) >	<([0.7,0.8],0.75), ([0.1,0.2],0.15), ([0.1,0.2],0.15) >	<([0.6,0.7],0.65), ([0.2,0.3],0.25), ([0.2,0.3],0.25) >	<([0.3,0.4],0.35), ([0.3,0.4],0.35), ([0.3,0.6],0.55) >
G_3	<([0.3,0.4],0.35), ([0.3,0.4],0.35), ([0.3,0.6],0.55) >	<([0.6,0.7],0.65), ([0.2,0.3],0.25), ([0.2,0.3],0.25) >	<([0.4,0.5],0.45), ([0.4,0.5],0.45), ([0.4,0.5],0.45) >	<([0.6,0.7],0.65), ([0.2,0.3],0.25), ([0.2,0.3],0.25) >
G_4	<([0.4,0.5],0.45), ([0.4,0.5],0.45), ([0.4,0.5],0.45) >	<([0.6,0.7],0.65), ([0.2,0.3],0.25), ([0.2,0.3],0.25) >	<([0.7,0.8],0.75), ([0.1,0.2],0.15), ([0.1,0.2],0.15) >	<([0.6,0.7],0.65), ([0.2,0.3],0.25), ([0.2,0.3],0.25) >

Step 2: Because the criteria in the assessment of service quality of crowdsourcing logistics platform are all benefit criteria, hence the normalized process can be omitted.

Step 3: Compute the weight of criteria. Firstly, we need to figure out the score matrix $SF = (sf_{i\lambda})_{m\times n}$ of the NSCFDM by Eq.(18). Then the Renyi entropy E_λ of score function matrix $SF =$

$(sf_{i\lambda})_{m\times n}$ can be computed by Eq.(19). After that, the objective weight ϖ_λ of criteria can be worked out by Eqs.(21)-(22) and listed as $\varpi_1 = 0.2487$, $\varpi_2 = 0.2554$, $\varpi_3 = 0.2500$, $\varpi_4 = 0.2460$, where the parameter β is set as 0.5.

Step 4: Compute the weighted sum measure (WSM) based on SCFAAWA operator by Eq. (23)

$$\rho_1 = \begin{Bmatrix} ([0.5938,0.6964], 0.6448), ([0.2002,0.3082], 0.2550), \\ ([0.2002,0.3082], 0.2550) \end{Bmatrix},$$

$$\rho_2 = \begin{Bmatrix} ([0.5843,0.6874], 0.6354), ([0.1851,0.2903], 0.2384), \\ ([0.1851,0.3208], 0.2664) \end{Bmatrix},$$

$$\rho_3 = \begin{Bmatrix} ([0.5023,0.6035], 0.5526), ([0.2631,0.3661], 0.3148), \\ ([0.2631,0.4050], 0.3523) \end{Bmatrix},$$

$$\rho_4 = \begin{Bmatrix} ([0.5942,0.6968], 0.6451), ([0.1998,0.3078], 0.2547), \\ ([0.1998,0.3078], 0.2547) \end{Bmatrix}.$$

Step 5: Compute the weighted product measure (WPM) based on SCFAAWG operator by Eq. (24)

$$\sigma_1 = \begin{Bmatrix} ([0.5633,0.6652], 0.6144), ([0.2002,0.3082], 0.2550), \\ ([0.2538,0.3483], 0.3004) \end{Bmatrix},$$

$$\sigma_2 = \begin{Bmatrix} ([0.5263,0.6311], 0.5790), ([0.1851,0.2903], 0.2384), \\ ([0.2123,0.3924], 0.3448) \end{Bmatrix},$$

$$\sigma_3 = \begin{Bmatrix} ([0.4563,0.5599], 0.5083), ([0.2631,0.3661], 0.3148), \\ ([0.2895,0.4534], 0.4043) \end{Bmatrix},$$

$$\sigma_4 = \begin{Bmatrix} ([0.5638,0.6657], 0.6148), ([0.1998,0.3078], 0.2547), \\ ([0.2533,0.3479], 0.3000) \end{Bmatrix}.$$

Step 6: Compute the score values of WSM φ_i and WPM ϕ_i by Eqs.(25)-(26)
$\varphi_1 = 0.5917$, $\varphi_2 = 0.5865$, $\varphi_3 = 0.5594$, $\varphi_4 = 0.5919$, $\phi_1 = 0.5783$, $\phi_2 = 0.5641$, $\phi_3 = 0.5503$, $\phi_4 = 0.5785$.

Step 7: Determine the comprehensive assessment value of each scheme by Eq.(27) wherein the parameter χ is set as 0.5, the outcomes are displayed as $\Xi_1 = 0.5850$, $\Xi_2 = 0.5753$, $\Xi_3 = 0.5548$, $\Xi_4 = 0.5852$. Hence, the ranking of CLP is $G_4 \succ G_1 \succ G_2 \succ G_3$.

5.2 Parameter discussions and analysis

In the proposed decision model, two parameters maybe cause influence for the final decision outcomes. Hence, we conduct the parameter discussions and analysis for validating the stability and robustness of the proposed approach. Firstly, we use diverse parameter values of χ in the proposed method, the corresponding results are listed in Table 4 and Figure 1. From the results, we can find that the ranking results attained by diverse values of parameter χ are consistency, which implies that the proposed method is stable for parameter χ.

Table 4
Decision outcomes based on diverse values of parameter χ

χ	G1	G2	G3	G4	Ranking
0.0	0.5783	0.5641	0.5503	0.5785	$G_4 \succ G_1 \succ G_2 \succ G_3$
0.1	0.5797	0.5663	0.5512	0.5798	$G_4 \succ G_1 \succ G_2 \succ G_3$
0.2	0.5810	0.5685	0.5521	0.5812	$G_4 \succ G_1 \succ G_2 \succ G_3$
0.3	0.5823	0.5708	0.5530	0.5825	$G_4 \succ G_1 \succ G_2 \succ G_3$
0.4	0.5837	0.5730	0.5539	0.5838	$G_4 \succ G_1 \succ G_2 \succ G_3$
0.5	0.5850	0.5753	0.5548	0.5852	$G_4 \succ G_1 \succ G_2 \succ G_3$
0.6	0.5864	0.5775	0.5557	0.5865	$G_4 \succ G_1 \succ G_2 \succ G_3$
0.7	0.5877	0.5798	0.5567	0.5879	$G_4 \succ G_1 \succ G_2 \succ G_3$
0.8	0.5890	0.5820	0.5576	0.5892	$G_4 \succ G_1 \succ G_2 \succ G_3$
0.9	0.5904	0.5842	0.5585	0.5905	$G_4 \succ G_1 \succ G_2 \succ G_3$
1.0	0.5917	0.5865	0.5594	0.5919	$G_4 \succ G_1 \succ G_2 \succ G_3$

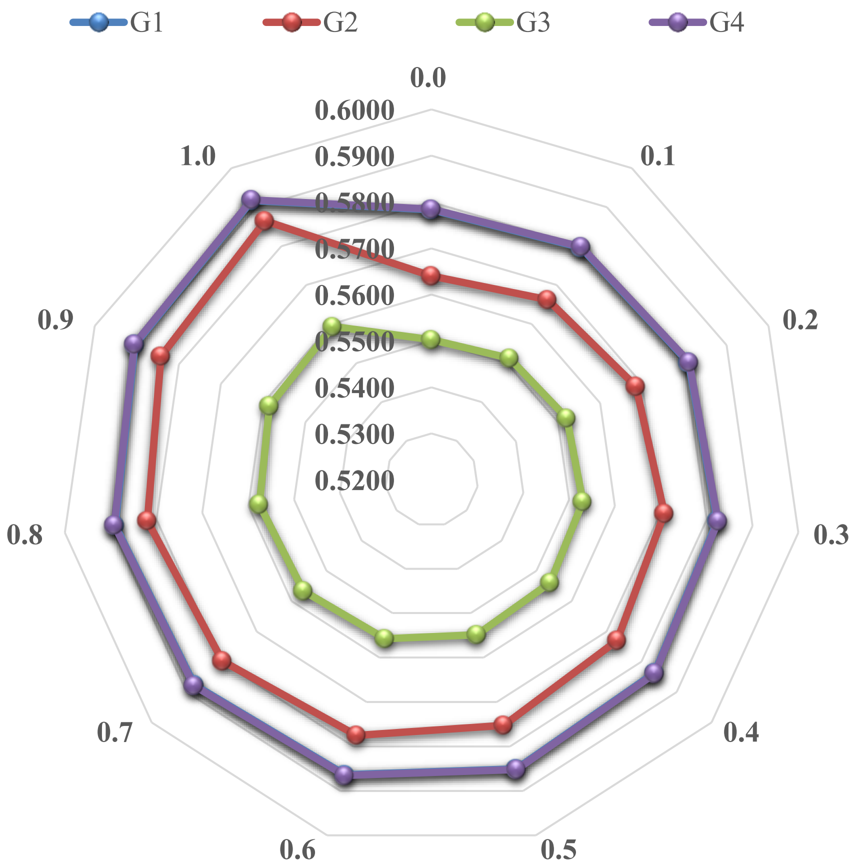
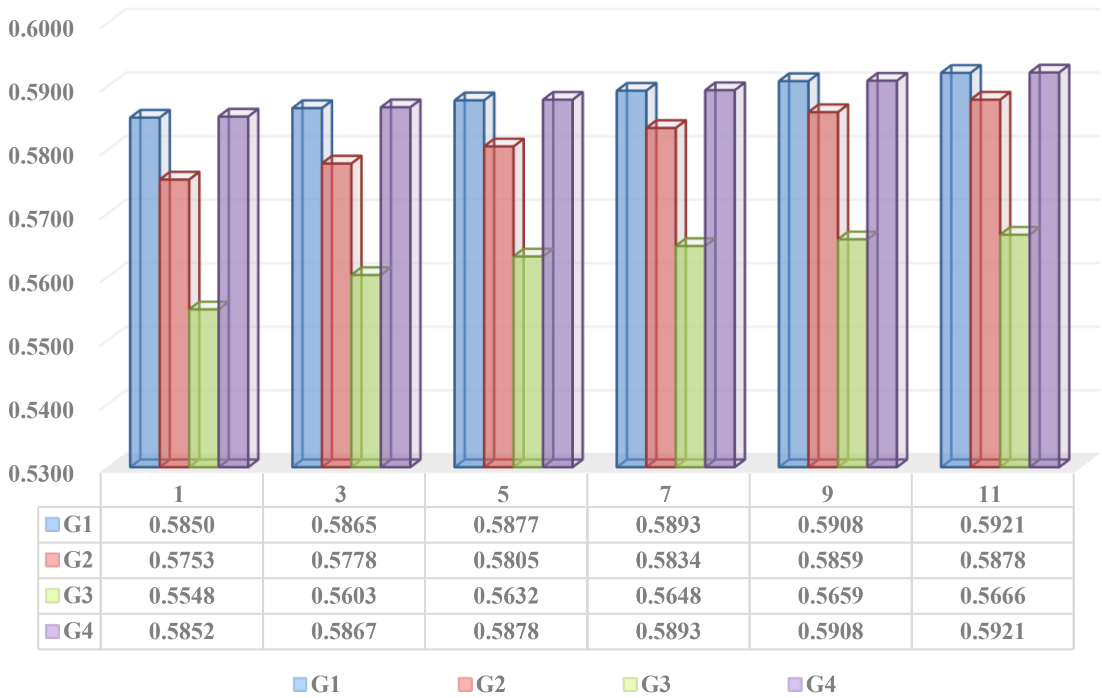


Fig. 1. Decision outcomes based on diverse values of parameter χ

Secondly, we discuss the influence of parameter ∂ in the proposed SCFAAWA and SCFAAWG operators on the proposed decision model, the decision outcomes based on different parameter values of are shown in Figure 2. From it, we can attain that the ranking of CLP is $G_4 \succ G_1 \succ G_2 \succ G_3$ when $\partial = 1,3,5$, while the ranking of CLP can be acquired as $G_4 \sim G_1 \succ G_2 \succ G_3$ when $\partial = 7,9,11$. Although it has a little difference, the optimal option is always G_4, which also reflects the stability of the proposed method.



	1	3	5	7	9	11
G1	0.5850	0.5865	0.5877	0.5893	0.5908	0.5921
G2	0.5753	0.5778	0.5805	0.5834	0.5859	0.5878
G3	0.5548	0.5603	0.5632	0.5648	0.5659	0.5666
G4	0.5852	0.5867	0.5878	0.5893	0.5908	0.5921

Fig. 2. Decision outcomes based on diverse values of parameter ∂

6. Conclusion

In this study, we propose a novel decision framework via combining the Renyi entropy weight method, WASPAS method and Aczel-Alsina operators under SCF environment to evaluate the service quality of crowdsourcing logistics platform. In light of the merits of SCFS in expressing uncertainty information, it is utilized to portray the fuzziness and uncertain information in the evaluation process. In order to construct the decision methodology, we define the SCF Aczel-Alsina operations and the proposed four aggregation operators to fuse SCF information. Then we propound the SCF Renyi entropy weight method to gain the weight of assessment criteria. In addition, we develop an improved WASPAS method based on the proposed SCF Aczel-Alsina aggregation operators. Finally, we validate the applicability of the proposed methodology by a case study. The sensitivity analysis is carried out to discuss the sensitivity of the proposed method in the process of decision analysis. The presented approach provides a flexible decision model for uncertain decision analysis. Future works should investigate the theorical research and applications under SCF environment, including aggregation operators based on diverse fusion functions and several information measures.

Conflicts of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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