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Diagnosis of Coronary Heart Disease Through Fuzzy Information Measures and Pattern Recognition-Based Segmentation and Localization in Computed Tomography Angiography

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ABSTRACT

Timely detection of coronary heart disease (CHD) is both significant and challenging. Advanced computational techniques such as similarity measures (SMs), entropy measures, and distance-measure-based segmentation can greatly enhance the precision, speed, and reliability of computed tomography angiography (CTA) image interpretation, thereby supporting more effective early detection and management of CHD. The concept of the interval-valued spherical fuzzy set (Iv-SFS) serves as a powerful tool for handling complex and ambiguous information, as it allows the representation of membership, abstinence, and non-membership values in the form of intervals. Building on the structure of Iv-SFS, this study introduces novel SMs, including interval-valued spherical fuzzy cosine SMs (Iv-SFCSM), Iv-SF cosine weighted SMs (Iv-SFCWSM), Iv-SF dice SMs (Iv-SFDSM), and Iv-SF dice weighted SMs (Iv-SFDWSM). Several axioms of SMs are demonstrated to verify the validity of the proposed measures. The developed SMs are then applied to solve a multi-attribute decision-making (MADM) problem for CHD diagnosis. Key features of the proposed approach are discussed, and comparative analyses with existing SMs highlight its advantages. The paper concludes with remarks underscoring the effectiveness and applicability of the proposed framework.

1. Introduction

The introduction section is based on the following subsection: a brief history of fuzzy set theory and its extensions, decision-making sciences and literature review, the role of similar measures in MADM, problem questions and motivation, and organization of proposed work.

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1.1. History of fuzzy set theory and their generalization

The delicacy of the crips set theory was covered by the fuzzy set (FS) theory. The thought of FS was first presented by Zadeh [1] by introducing the concept of membership valued (MV) $\mathcal{M}$ under the range of closed interval $\mathcal{M} \in [0, 1]$. In addition, the generalized format of FS called internal-valued FS (Iv-FS) we given by Zadeh [2], where $\mathcal{M}^l \in [0, 1]$, and $\mathcal{M}^u \in [0, 1]$. This idea got more popularity than ordinary FS, because it has an ability to express human idea in term of intervals, which make more reliability in decision making problems. The main disadvantage of FS theory is that, it does not provide information about the non-membership value (NV) $\underline{đ}$. So, to overcome this gap Atanassov [3] derived the idea of intuitionistic fuzzy set (IFS) with addition of NV in FS, where the sum of MV and NV lies between the $[0, 1]$ such as $0 \leq \mathcal{M} + \underline{đ} \leq 1$. In addition, Atanassov and Gargov [4] generalized the theory of IFS and presented the novel concept of interval-valued IFS (Iv-IFS) where $\mathcal{M}^l \in [0, 1], \mathcal{M}^u \in [0, 1], \underline{đ}^l \in [0, 1]$ and $\underline{đ}^l \in [0, 1]$. But for many situations, all above discussed concepts are failed to handle information if one additional parametric value for accurate assessment of data called abstinence value (AV). The theory of Picture FS (PFS) was developed by [5, p. 201] by making the addition of AV in IFS under the range of interval $[0, 1]$ like $0 \leq \mathcal{M} + Ꞥ + \underline{đ} \leq 1$. The generalization of PFS into the interval-valued PFS to express the MV, NMV, and AV in terms of the interval is proposed by [6]. However, there are many situations present in decision-making sciences where the sum of fuzzy data exceeds by closed interval $[0, 1]$, the suggested PFS failed to aggregate information. Mahmood *et al.*, [7] introduced the SFS framework to reduce this shortcoming, where the sum of the MV, AV, and NV is always under the zero and one interval range. Furthermore, the concept of Iv-SFS for the assessment of fuzzy information was provided by Duleba *et al.*, [8].

1.2. Decision-making sciences and literature review

The thought of decision-making sciences is a powerful tool for deep assessment of large amounts of information. The idea of MADM is a useful concept in which decision-makers try to find the best alternative from a list of finite alternatives under the consideration of finite multiple attributes. MADM is valuable in pattern reorganization, cluster analysis, the medical field, engineering, and data sciences. By inspiring from these keynote features in different fields, many mathematicians worked a lot in decision-making science and proposed many approached for solving MADM problems based on FS theory. For example, Khan *et al.*, [9] solved the decision-making problem using the concept of complex q-rung orthopair FS (q-ROFS), and Zhang *et al.*, [10] proposed the MADM approach for the assessment of electric cars based on the q-ROFS theory. Ullah *et al.*, [11] developed the MADM technique based on the t-spherical FS theory. The MADM methodology based on the Einstein operations and t-spherical FS was defined by Munir *et al.*, [12]. Ashraf *et al.*, [13] derived MADM theory based on the SFS and the solution of MADM problems based on the Picture FS (PFS) was given by Ashraf *et al.*, [14].

1.3. Role of similarity measures in MADM

SMs are essential for comparing options based on several criteria in the MADM approach. Frequently using distance-based techniques like cosine similarity, dice similarity, grey similarity, and Euclidean distance assists in quantifying how near each alternative is to the ideal or intended solution. These SMs rank and compare alternatives to the ideal or anti-ideal solutions. Also simplify the decision-making process, SMs can help with clustering options, handling fuzziness or uncertainty in decision data, and aggregating criteria. Based on these significant features, many mathematicians proposed multiple SMs for different fuzzy frameworks, such as SMs based on the t-spherical FS information proposed by Ullah *et al.*, [15] and SMs based on the complex bipolar FS theory diagnosed by Mahmood and Rehman [16]. Rafiq *et al.*, [17] developed some cosine SMs based on the SFS information and SMs proposed by utilizing Ali and Mahmood's complex neutrosophic FS theory [18]. The complex vector hybrid similarity measure for complex hesitant fuzzy information derived by Mahmood *et al.*, [19] and the interval-valued PFS based similarity measure proposed by Liu *et al.*, [20].

1.4. Importance of weight vectors

Weight vectors play an essential role in MADM because they show how important various criteria or qualities are with one another. Decision-makers can prioritize particular features of the options under consideration by giving each criterion the appropriate weight, guaranteeing that more important considerations will have a bigger impact on the choice made in the end. So, the analytical hierarchy process (AHP) is one of the best tools for weight calculation for the MADM technique. The AHP can assess the hierarchy's elements at each stage in pairs. Here, two choices are contrasted based on their effects on a higher-ranking element in the hierarchy. For these comparisons, Saaty [21] developed a scale where one represents equal relevance and nine indicates that one attribute is highly significant compared to another. Many mathematicians worked a lot in the decision-making field by using multiple fuzzy frameworks such as Mahmood *et al.,* [22] proposed the AHP model under the PFS environment, and Acar *et al.,* [23] developed the AHP-based MADM approach for the stationary hydrogen storage problem. Farooq [24] proposed the AHP for the Pythagorean fuzzy framework, and Kahraman [25] presented multiple fuzzy extensions based on AHP for MADM problems.

1.5. Problem questions and motivation

SMs are very important in detecting heart disease and all medical fields. For example, detecting CHD using FS-based SMs enables us to treat patients on time better than the traditional way (Doctors just believe their experiences do not use medical test reports generated from laboratories). Nowadays, we have computed tomography angiography (CTA) machines to detect heart disease. The CTA machine improves the effectiveness and accuracy of identifying cardiac issues. In this proposed work, we have addressed the following questions:

i. i. When the patient's medical history is ambiguous or lacking, how may Iv-SFS information methods be applied to increase the accuracy of CHD diagnosis?

ii. ii. When identifying CHD, how do IV-SFS theory-based SMs improve decision-making?

iii. iii. How may segmentation based on pattern recognition lessen human error in CTA image analysis to diagnose CHD?

By motivating from the previous literature review, it is noticed that many SMs are already present for the investigation of fuzzy, but when information is provided in the form of Iv-SFS, there is no structure present to handle the Iv-SFS framework. So, in this article, we aimed to propose Iv-SFCSM, Iv-SFCWSM, Iv-SFDSM, and Iv-SFDWSM by utilizing the spherical FS (SFS) theory. We also discussed some necessary axioms of SMs.

1.6. Organization of proposed work

The remaining manuscript is managed as follows: some necessary definitions discussed in preliminary Section 2. Newly proposed SMs, including proofs discussed in Section 3. The calculation of weight vectors by using the AHP method is discussed in Section 4. The application for detection of CHD and assessment of CTA is presented in Section 5 and numerical example by using MADM is provided in Section 6. Section 7 offers the comparative analysis of the presented approach for investigation of the effectiveness of developed SMs. Some solid conclusions are discussed in Section 8.

2. Preliminaries

This section presents some basic notions like FS, interval-valued FS, IFS and Iv-IFS, SFS, and Iv-SFS, which will help understand further discussions.

The idea of FS was a big revolution in the decision-making field because it gives the freedom to decision make to express their thoughts in terms of MV. Further, all manuscript $\mathcal{L}$ denotes the universal set.

Definition 1 [1]. A FS Ò on $\mathcal{L}$ is defined as follows:

$$Ò = \{(ⱡ, ʍ(ⱡ)) \mid ⱡ \in \mathcal{L}\} \tag{1}$$

Where ʍ denoted as MV and $ʍ: \mathcal{L} \to [0,1]$. For convenience, the $ʍ(ⱡ)$ called fuzzy value (FV).

Definition 2 [2]. An Iv-FS Ò on $\mathcal{L}$ is defined as follows:

$$Ò = \{(ⱡ, ʍ_r(ⱡ)) \mid ⱡ \in \mathcal{L}\} \tag{2}$$

Where $ʍ_r$ denoted as MV and $ʍ_r: \mathcal{L} \to [0,1]$. For convenience, the $ʍ_r(ⱡ) = [ʍ_r^l, ʍ_r^u]$ called interval-valued fuzzy value (Iv-FV).

Definition 3 [3]. An IFS Ò on $\mathcal{L}$ is defined as follows:

$$Ò = \{(ⱡ, ʍ(ⱡ), đ(ⱡ)) \mid ⱡ \in \mathcal{L}\} \tag{3}$$

Where ʍ denoted as MV and đ denoted as NMV, $ʍ: \mathcal{L} \to [0,1]$, $đ: \mathcal{L} \to [0,1]$. For convenience, the duplet $(ʍ(ⱡ), đ(ⱡ))$ called intuitionistic fuzzy values (IFV).

Definition 4 [4]. An Iv-IFS Ò on $\mathcal{L}$ is defined as follows:

$$Ò = \{(ⱡ, ʍ_r(ⱡ), đ_r(ⱡ)) \mid ⱡ \in \mathcal{L}\} \tag{4}$$

Where $ʍ_r = [ʍ_r^l, ʍ_r^u]$ denoted as MV and $đ_r = [đ_r^l, đ_r^u]$ denoted as NMV and $ʍ_r: \mathcal{L} \to [0,1]$, $đ_r: \mathcal{L} \to [0,1]$. For convenience, the $([ʍ_r^l, ʍ_r^u], [đ_r^l, đ_r^u])$ called interval-valued IFV (Iv-IFV).

Definition 5. An SFS Ò on $\mathcal{L}$ is defined as follows:

$$Ò = \{(ⱡ, ʍ(ⱡ), Ⱥ(ⱡ), đ(ⱡ)) \mid ⱡ \in \mathcal{L}\} \tag{5}$$

Where ʍ, denoted as MV, Ⱥ denoted as AV, and đ denoted as NMV, $ʍ: \mathcal{L} \to [0,1]$, $Ⱥ: \mathcal{L} \to [0,1]$, $đ: \mathcal{L} \to [0,1]$. For convenience, the duplet $(ʍ(ⱡ), Ⱥ(ⱡ), đ(ⱡ))$ called spherical fuzzy values (SFV).

Definition 6 [8]. An Iv-SFS Ò on $\mathcal{L}$ is defined as follows:

$$Ò = \{(ⱡ, ʍ_r(ⱡ), Ⱥ_r(ⱡ), đ_r(ⱡ)) \mid ⱡ \in \mathcal{L}\} \tag{6}$$

Where $ʍ_r = [ʍ_r^l, ʍ_r^u]$ denoted as MV, $Ⱥ_r = [Ⱥ_r^l, Ⱥ_r^u]$ denoted as AV and $đ_r = [đ_r^l, đ_r^u]$ denoted as NMV and $ʍ_r: \mathcal{L} \to [0,1]$, $Ⱥ_r: \mathcal{L} \to [0,1]$, $đ_r: \mathcal{L} \to [0,1]$. For convenience, the $([ʍ_r^l, ʍ_r^u], [Ⱥ_r^l, Ⱥ_r^u], [đ_r^l, đ_r^u])$ called interval-valued SFV (Iv-SFV).

3. Proposed similarity measures based on Iv-SFS information

In this section, we will define some new similarity measures like $Iv - SFCSM^1, Iv - SFCWSM^1, Iv - SFCWSM^2, Iv - SFDSM$, and Iv-SFDWSM for the solution of MADM problems.

3.1. Some cosines similarity measures

This subsection discussed the newly proposed $Iv - SFCSM^1, Iv - SFCWSM^1, Iv - SFCWSM^2, Iv - SFDSM$ under the Iv-SFS framework, and some basic axioms of proposed SMs are also investigated in this segment.

Definition 7. Consider two Iv-SFSVs $R_1 = ([ʍ_1^l, ʍ_1^u], [Ⱥ_1^l, Ⱥ_1^u], [đ_1^l, đ_1^u])$, $R_2 = ([ʍ_2^l, ʍ_2^u], [Ⱥ_2^l, Ⱥ_2^u], [đ_2^l, đ_2^u])$ and $k \geq 1$, the two cosine SMs between and R_1 and R_2 given as follows:

$$Iv - SFCSM^1(R_1, R_2) =$$

$$\frac{1}{k}\sum_{s=1}^{k}\frac{\text{ʍ}_1^l\text{ʍ}_2^l+\text{Ⱥ}_1^l\text{Ⱥ}_2^l+\underline{\text{đ}}_1^l\underline{\text{đ}}_2^l+\text{ʍ}_1^u\text{ʍ}_2^u+\text{Ⱥ}_1^u\text{Ⱥ}_2^u+\underline{\text{đ}}_1^u\underline{\text{đ}}_2^u}{\sqrt{\text{ʍ}^2{}_1^l+\text{Ⱥ}^2{}_1^l+\underline{\text{đ}}^2{}_1^l+\text{ʍ}^2{}_1^u+\text{Ⱥ}^2{}_1^u+\underline{\text{đ}}^2{}_1^u}\cdot\sqrt{\text{ʍ}^2{}_2^l+\text{Ⱥ}^2{}_2^l+\underline{\text{đ}}^2{}_2^l+\text{ʍ}^2{}_2^u+\text{Ⱥ}^2{}_2^u+\underline{\text{đ}}^2{}_2^u}} \tag{7}$$

Theorem 1. Consider two Iv-SFSVs $R_1=\left([\text{ʍ}_1^l,\text{ʍ}_1^u],[\text{Ⱥ}_1^l,\text{Ⱥ}_1^u],[\underline{\text{đ}}_1^l,\underline{\text{đ}}_1^u]\right), R_2=\left([\text{ʍ}_2^l,\text{ʍ}_2^u],[\text{Ⱥ}_2^l,\text{Ⱥ}_2^u],[\underline{\text{đ}}_2^l,\underline{\text{đ}}_2^u]\right)$ and $k\geq 1$, then holds the following axioms:

i. $0\leq Iv-SFCSM^1(R_1,R_2)\leq 1$
ii. $Iv-SFCSM^1(R_1,R_2)=Iv-SFCSM^1(R_2,R_1)$
iii. For $R_1=R_2, Iv-SFCSM^1(R_1,R_2)=1$

Proof:

As MV, AVs, and NMV of all Iv-SFSVs belong from the closed interval $[0,1]$, then it is obvious that $Iv-SFCSM^1(R_1,R_2)\in[0,1]$.

i. Satisfy trivially.
ii. If we take $R_1=R_2$ the is obviously $\text{ʍ}_1^l=\text{ʍ}_2^l, \text{Ⱥ}_1^l=\text{Ⱥ}_2^l, \underline{\text{đ}}_1^l=\underline{\text{đ}}_2^l, \text{ʍ}_1^u=\text{ʍ}_2^u, \text{Ⱥ}_1^u=\text{Ⱥ}_2^u, \underline{\text{đ}}_1^u=\underline{\text{đ}}_2^u$

Then

$$Iv-SFCSM^1(R_1,R_2)=\frac{1}{k}\sum_{s=1}^{k}\frac{\text{ʍ}^2{}_1^l+\text{Ⱥ}^2{}_1^l+\underline{\text{đ}}^2{}_1^l+\text{ʍ}^2{}_1^u+\text{Ⱥ}^2{}_1^u+\underline{\text{đ}}^2{}_1^u}{\text{ʍ}^2{}_1^l+\text{Ⱥ}^2{}_1^l+\underline{\text{đ}}^2{}_1^l+\text{ʍ}^2{}_1^u+\text{Ⱥ}^2{}_1^u+\underline{\text{đ}}^2{}_1^u}$$

$$Iv-SFCSM^1(R_1,R_2)=\frac{1}{k}\sum_{s=1}^{k}1=\frac{1}{k}.k=1$$

Definition 8. Consider two Iv-SFSVs $R_1=\left([\text{ʍ}_1^l,\text{ʍ}_1^u],[\text{Ⱥ}_1^l,\text{Ⱥ}_1^u],[\underline{\text{đ}}_1^l,\underline{\text{đ}}_1^u]\right), R_2=\left([\text{ʍ}_2^l,\text{ʍ}_2^u],[\text{Ⱥ}_2^l,\text{Ⱥ}_2^u],[\underline{\text{đ}}_2^l,\underline{\text{đ}}_2^u]\right)$ and weight vectors $\sum_{s=1}^{k}\ddot{\text{W}}_k=1$ and $(\ddot{\text{W}}_1,\ddot{\text{W}}_2,\dots,\ddot{\text{W}}_k)\in[0,1]$, the two cosine weighted SMs between and R_1 and R_2 given as follows:

$$Iv-SFCWSM^1(R_1,R_2)=\sum_{s=1}^{k}\ddot{\text{W}}_k\frac{\text{ʍ}_1^l\text{ʍ}_2^l+\text{Ⱥ}_1^l\text{Ⱥ}_2^l+\underline{\text{đ}}_1^l\underline{\text{đ}}_2^l+\text{ʍ}_1^u\text{ʍ}_2^u+\text{Ⱥ}_1^u\text{Ⱥ}_2^u+\underline{\text{đ}}_1^u\underline{\text{đ}}_2^u}{\sqrt{\text{ʍ}^2{}_1^l+\text{Ⱥ}^2{}_1^l+\underline{\text{đ}}^2{}_1^l+\text{ʍ}^2{}_1^u+\text{Ⱥ}^2{}_1^u+\underline{\text{đ}}^2{}_1^u}\cdot\sqrt{\text{ʍ}^2{}_2^l+\text{Ⱥ}^2{}_2^l+\underline{\text{đ}}^2{}_2^l+\text{ʍ}^2{}_2^u+\text{Ⱥ}^2{}_2^u+\underline{\text{đ}}^2{}_2^u}} \tag{8}$$

Theorem 2. Consider two Iv-SFSVs $R_1=\left([\text{ʍ}_1^l,\text{ʍ}_1^u],[\text{Ⱥ}_1^l,\text{Ⱥ}_1^u],[\underline{\text{đ}}_1^l,\underline{\text{đ}}_1^u]\right), R_2=\left([\text{ʍ}_2^l,\text{ʍ}_2^u],[\text{Ⱥ}_2^l,\text{Ⱥ}_2^u],[\underline{\text{đ}}_2^l,\underline{\text{đ}}_2^u]\right)$ and weight vectors $\sum_{s=1}^{k}\ddot{\text{W}}_k=1$ and $(\ddot{\text{W}}_1,\ddot{\text{W}}_2,\dots,\ddot{\text{W}}_k)\in[0,1]$, then holds the following axioms:

iii. $0\leq Iv-SFCWSM^1(R_1,R_2)\leq 1$
iv. $Iv-SFCWSM^1(R_1,R_2)=Iv-SFCWSM^2(R_2,R_1)$
v. For $R_1=R_2, Iv-SFCWSM^1(R_1,R_2)=1$

Proof:

As MV, AV, and NMV of all Iv-SFSVs belong to the closed interval $[0,1]$, then it is obvious that $Iv-SFCSM^1(R_1,R_2)\in[0,1]$.

i. Satisfy trivially.

If we take $R_1=R_2$ the is obviously $\text{ʍ}_1^l=\text{ʍ}_2^l, \text{Ⱥ}_1^l=\text{Ⱥ}_2^l, \underline{\text{đ}}_1^l=\underline{\text{đ}}_2^l, \text{ʍ}_1^u=\text{ʍ}_2^u, \text{Ⱥ}_1^u=\text{Ⱥ}_2^u, \underline{\text{đ}}_1^u=\underline{\text{đ}}_2^u$

Then

$$Iv-SFCWSM^1(R_1,R_2)=\sum_{s=1}^{k}\ddot{\text{W}}_k\frac{\text{ʍ}^2{}_1^l+\text{Ⱥ}^2{}_1^l+\underline{\text{đ}}^2{}_1^l+\text{ʍ}^2{}_1^u+\text{Ⱥ}^2{}_1^u+\underline{\text{đ}}^2{}_1^u}{\text{ʍ}^2{}_1^l+\text{Ⱥ}^2{}_1^l+\underline{\text{đ}}^2{}_1^l+\text{ʍ}^2{}_1^u+\text{Ⱥ}^2{}_1^u+\underline{\text{đ}}^2{}_1^u}$$

$$Iv-SFCSM^{1}(R_1,R_2)=\sum_{s=1}^{k}\ddot{W}_k=1$$

Definition 9. Consider two Iv-SFSVs $R_1=([ʍ_1^l,ʍ_1^u],[Ⱥ_1^l,Ⱥ_1^u],[đ_1^l,đ_1^u]),R_2=([ʍ_2^l,ʍ_2^u],[Ⱥ_2^l,Ⱥ_2^u],[đ_2^l,đ_2^u])$, $k\geq 1$, and weight vectors $\sum_{s=1}^{k}\ddot{W}_k=1$ and $(\ddot{W}_1,\ddot{W}_2,\ldots,\ddot{W}_k)\in[0,1]$, the two cosine SMs between and R_1 and R_2 including refusal $(r^l,r^u)\in[0,1]$ between MV, AV, and NMV given as follows:

$$Iv-SFCSM^{2}(R_1,R_2)=$$
$$\frac{1}{k}\sum_{s=1}^{k}\frac{\begin{matrix}ʍ_1^lʍ_2^l+Ⱥ_1^lȺ_2^l+đ_1^lđ_2^l+\\ r_1^lr_2^l+ʍ_1^uʍ_2^u+Ⱥ_1^uȺ_2^u+đ_1^uđ_2^u+r_1^ur_2^u\end{matrix}}{\sqrt{\begin{matrix}ʍ^2{}_1^l+Ⱥ^2{}_1^l+đ^2{}_1^l+\\ r^2{}_1^l+ʍ^2{}_1^u+Ⱥ^2{}_1^u+đ^2{}_1^u+r^2{}_1^u\end{matrix}}\cdot\sqrt{\begin{matrix}ʍ^2{}_2^l+Ⱥ^2{}_2^l+đ^2{}_2^l+\\ r^2{}_2^l+ʍ^2{}_2^u+Ⱥ^2{}_2^u+đ^2{}_2^u+r^2{}_2^u\end{matrix}}} \tag{9}$$

and

$$Iv-SFCSM^{2}(R_1,R_2)=$$
$$\sum_{s=1}^{k}\ddot{W}_k\frac{\begin{matrix}ʍ_1^lʍ_2^l+Ⱥ_1^lȺ_2^l+đ_1^lđ_2^l+\\ r_1^lr_2^l+ʍ_1^uʍ_2^u+Ⱥ_1^uȺ_2^u\\ +đ_1^uđ_2^u+r_1^ur_2^u\end{matrix}}{\sqrt{\begin{matrix}ʍ^2{}_1^l+Ⱥ^2{}_1^l+đ^2{}_1^l\\ +r^2{}_1^l+ʍ^2{}_1^u+Ⱥ^2{}_1^u+đ^2{}_1^u+r^2{}_1^u\end{matrix}}\cdot\sqrt{\begin{matrix}ʍ^2{}_2^l+Ⱥ^2{}_2^l+đ^2{}_2^l\\ +r^2{}_2^l+ʍ^2{}_2^u+Ⱥ^2{}_2^u+đ^2{}_2^u+r^2{}_2^u\end{matrix}}} \tag{10}$$

Theorem 3. Consider two Iv-SFSVs $R_1=([ʍ_1^l,ʍ_1^u],[Ⱥ_1^l,Ⱥ_1^u],[đ_1^l,đ_1^u]),R_2=([ʍ_2^l,ʍ_2^u],[Ⱥ_2^l,Ⱥ_2^u],[đ_2^l,đ_2^u])$ and $k\geq 1$, based on four functions, holds the following axioms:

i. $0\leq Iv-SFCSM^{2}(R_1,R_2)\leq 1$
ii. $Iv-SFCSM^{2}(R_1,R_2)=Iv-SFCSM^{2}(R_2,R_1)$
iii. For $R_1=R_2, Iv-SFCSM^{2}(R_1,R_2)=1$
iv. $0\leq Iv-SFWCSM^{2}(R_1,R_2)\leq 1$
v. $Iv-SFCWSM^{2}(R_1,R_2)=Iv-SFCWSM^{2}(R_2,R_1)$
vi. For $R_1=R_2, Iv-SFCWSM^{2}(R_1,R_2)=1$

Proof:

The proofs of the above-following theorem are the same as Theorem 1 and 2.

3.2. Some dice similarity measures

This subsection discussed the newly proposed $Iv-SFDSM^{1}, Iv-SFDWSM^{1}, Iv-SFDWSM^{2}, Iv-SFDSM$ for Iv-SFS-based information and some desirable axioms of developed SMs investigated in this segment.

Definition 10. Consider two Iv-SFSVs $R_1=([ʍ_1^l,ʍ_1^u],[Ⱥ_1^l,Ⱥ_1^u],[đ_1^l,đ_1^u]),R_2=([ʍ_2^l,ʍ_2^u],[Ⱥ_2^l,Ⱥ_2^u],[đ_2^l,đ_2^u])$, $k\geq 1$, the two dice SMs between and R_1 and R_2 including refusal $(r^l,r^u)\in[0,1]$ between MV, AV, and NMV given as follows:

$$Iv-SFDSM^{1}(R_1,R_2)=$$
$$\frac{1}{k}\sum_{s=1}^{k}\frac{2\begin{pmatrix}ʍ_1^lʍ_2^l+Ⱥ_1^lȺ_2^l+đ_1^lđ_2^l+\\ ʍ_1^uʍ_2^u+Ⱥ_1^uȺ_2^u+đ_1^uđ_2^u\end{pmatrix}}{\begin{pmatrix}ʍ^2{}_1^l+Ⱥ^2{}_1^l+đ^2{}_1^l+\\ ʍ^2{}_1^u+Ⱥ^2{}_1^u+đ^2{}_1^u\end{pmatrix}+\begin{pmatrix}ʍ^2{}_2^l+Ⱥ^2{}_2^l+đ^2{}_2^l+\\ ʍ^2{}_2^u+Ⱥ^2{}_2^u+đ^2{}_2^u\end{pmatrix}} \tag{11}$$

$$Iv-SFDSM^{2}(R_1,R_2)=$$

$$\frac{1}{k}\sum_{s=1}^{k}\frac{2\begin{pmatrix} ʍ_1^l ʍ_2^l+Ⱥ_1^l Ⱥ_2^l+\underline{đ}_1^l \underline{đ}_2^l+\\ r_1^l r_2^l+ʍ_1^u ʍ_2^u+\\ Ⱥ_1^u Ⱥ_2^u+\underline{đ}_1^u \underline{đ}_2^u+r_1^u r_2^u\end{pmatrix}}{\begin{pmatrix} {ʍ^2}_1^l+{Ⱥ^2}_1^l+{\underline{đ}^2}_1^l+\\ {r^2}_1^l+{ʍ^2}_1^u+{Ⱥ^2}_1^u+{\underline{đ}^2}_1^u+{r^2}_1^u\end{pmatrix}+\begin{pmatrix} {ʍ^2}_2^l+{Ⱥ^2}_2^l+{\underline{đ}^2}_2^l\\ +{r^2}_2^l+{ʍ^2}_2^u+{Ⱥ^2}_2^u+{\underline{đ}^2}_2^u+{r^2}_2^u\end{pmatrix}} \tag{12}$$

$$Iv-SFDSM^2(R_1,R_2)=$$

$$\frac{1}{k}\sum_{s=1}^{k}\frac{2\begin{pmatrix} ʍ_1^l ʍ_2^l+Ⱥ_1^l Ⱥ_2^l+\underline{đ}_1^l \underline{đ}_2^l+\\ r_1^l r_2^l+ʍ_1^u ʍ_2^u+\\ Ⱥ_1^u Ⱥ_2^u+\underline{đ}_1^u \underline{đ}_2^u+r_1^u r_2^u\end{pmatrix}}{\begin{pmatrix} {ʍ^2}_1^l+{Ⱥ^2}_1^l+\\ {\underline{đ}^2}_1^l+{r^2}_1^l+{ʍ^2}_1^u\\ +{Ⱥ^2}_1^u+{\underline{đ}^2}_1^u+{r^2}_1^u\end{pmatrix}+\begin{pmatrix} {ʍ^2}_2^l+{Ⱥ^2}_2^l+\\ {\underline{đ}^2}_2^l+{r^2}_2^l+{ʍ^2}_2^u\\ +{Ⱥ^2}_2^u+{\underline{đ}^2}_2^u+{r^2}_2^u\end{pmatrix}} \tag{13}$$

$$Iv-SFDSM^3(R_1,R_2)=$$

$$\frac{\sum_{s=1}^{k}2\begin{pmatrix} ʍ_1^l ʍ_2^l+Ⱥ_1^l Ⱥ_2^l+\underline{đ}_1^l \underline{đ}_2^l+\\ ʍ_1^u ʍ_2^u+Ⱥ_1^u Ⱥ_2^u+\underline{đ}_1^u \underline{đ}_2^u\end{pmatrix}}{\sum_{s=1}^{k}\begin{pmatrix} {ʍ^2}_1^l+{Ⱥ^2}_1^l+\\ {\underline{đ}^2}_1^l+{ʍ^2}_1^u+{Ⱥ^2}_1^u+{\underline{đ}^2}_1^u\end{pmatrix}+\sum_{s=1}^{k}\begin{pmatrix} {ʍ^2}_2^l+{Ⱥ^2}_2^l\\ +{\underline{đ}^2}_2^l+{ʍ^2}_2^u+{Ⱥ^2}_2^u+{\underline{đ}^2}_2^u\end{pmatrix}} \tag{14}$$

$$Iv-SFDSM^4(R_1,R_2)=$$

$$\frac{\sum_{s=1}^{k}2\begin{pmatrix} ʍ_1^l ʍ_2^l+Ⱥ_1^l Ⱥ_2^l\\ +\underline{đ}_1^l \underline{đ}_2^l+\\ r_1^l r_2^l+ʍ_1^u ʍ_2^u+\\ Ⱥ_1^u Ⱥ_2^u+\underline{đ}_1^u \underline{đ}_2^u+r_1^u r_2^u\end{pmatrix}}{\sum_{s=1}^{k}\begin{pmatrix} {ʍ^2}_1^l+{Ⱥ^2}_1^l+{\underline{đ}^2}_1^l\\ +{r^2}_1^l+{ʍ^2}_1^u+{Ⱥ^2}_1^u+{\underline{đ}^2}_1^u+{r^2}_1^u\end{pmatrix}+\sum_{s=1}^{k}\begin{pmatrix} {ʍ^2}_2^l+{Ⱥ^2}_2^l+{\underline{đ}^2}_2^l\\ +{r^2}_2^l+{ʍ^2}_2^u+{Ⱥ^2}_2^u+{\underline{đ}^2}_2^u+{r^2}_2^u\end{pmatrix}} \tag{15}$$

Theorem 4. Consider two Iv-SFSVs $R_1=\left([ʍ_1^l,ʍ_1^u],[Ⱥ_1^l,Ⱥ_1^u],[\underline{đ}_1^l,\underline{đ}_1^u]\right), R_2=\left([ʍ_2^l,ʍ_2^u],[Ⱥ_2^l,Ⱥ_2^u],[\underline{đ}_2^l,\underline{đ}_2^u]\right)$ and $k\geq 1$, then holds the following axions for $P=1,2,3,4$

i. $0\leq\ Iv-SFDSM^P(R_1,R_2)\leq 1$
ii. $Iv-SFCSM^P(R_1,R_2)=Iv-SFCSM^P(R_2,R_1)$
iii. For $R_1=R_2, Iv-SFCSM^1(R_1,R_2)=1$

Let us assume the $R_1\subseteq R_2\subseteq R_3$, then the $Iv-SFDSM^P(R_1,R_3)\leq Iv-SFDSM^P(R_1,R_2)\leq Iv-SFDSM^P(R_1,R_3)$.

Proof:

i. As MV, AVs, and NMV of all Iv-SFSVs belong from the closed interval $[0,1]$, then it is obvious that $Iv-SFDSM^1(R_1,R_2)\in[0,1]$.
ii. Satisfy trivially.
iii. If we take $R_1=R_2$ the is obviously $ʍ_1^l=ʍ_2^l, Ⱥ_1^l=Ⱥ_2^l, \underline{đ}_1^l=\underline{đ}_2^l, ʍ_1^u=ʍ_2^u, Ⱥ_1^u=Ⱥ_2^u, \underline{đ}_1^u=\underline{đ}_2^u$

Then

$$Iv-SFDSM^1(R_1,R_2)=\frac{1}{k}\sum_{s=1}^{k}\frac{2\left({ʍ^2}_1^l+{Ⱥ^2}_1^l+{\underline{đ}^2}_1^l+{ʍ^2}_1^u+{Ⱥ^2}_1^u+{\underline{đ}^2}_1^u\right)}{\left({ʍ^2}_1^l+{Ⱥ^2}_1^l+{\underline{đ}^2}_1^l+{ʍ^2}_1^u+{Ⱥ^2}_1^u+{\underline{đ}^2}_1^u\right)+\left({ʍ^2}_1^l+{Ⱥ^2}_1^l+{\underline{đ}^2}_1^l+{ʍ^2}_1^u+{Ⱥ^2}_1^u+{\underline{đ}^2}_1^u\right)}$$

$$Iv-SFDSM^1(R_1,R_2)=\frac{1}{k}\sum_{s=1}^{k}1=\frac{1}{k}.k=1$$

Definition 11. Consider two Iv-SFSVs $R_1=\left([ʍ_1^l,ʍ_1^u],[Ⱥ_1^l,Ⱥ_1^u],[\underline{đ}_1^l,\underline{đ}_1^u]\right), R_2=\left([ʍ_2^l,ʍ_2^u],[Ⱥ_2^l,Ⱥ_2^u],[\underline{đ}_2^l,\underline{đ}_2^u]\right)$ *and weight vectors* $\sum_{s=1}^{k}\bar{W}_k=1$ *and* $(\bar{W}_1,\bar{W}_2,\dots,\bar{W}_k)\in[0,1]$*, the two dice weighted SMs between and* R_1 *and* R_2 *given as follows:*

$$Iv-SFDSM^1(R_1,R_2)=$$

$$\frac{1}{k}\sum_{s=1}^{k}\ddot{W}_k \frac{2\begin{pmatrix} \text{ʍ}_1^l\text{ʍ}_2^l+\text{Ⱥ}_1^l\text{Ⱥ}_2^l \\ +\underline{\text{đ}}_1^l\underline{\text{đ}}_2^l+ \\ \text{ʍ}_1^u\text{ʍ}_2^u+\text{Ⱥ}_1^u\text{Ⱥ}_2^u+\underline{\text{đ}}_1^u\underline{\text{đ}}_2^u \end{pmatrix}}{\begin{pmatrix} \text{ʍ}^2{}_1^l+\text{Ⱥ}^2{}_1^l+\underline{\text{đ}}^2{}_1^l \\ +\text{ʍ}^2{}_1^u+\text{Ⱥ}^2{}_1^u+\underline{\text{đ}}^2{}_1^u \end{pmatrix}+\begin{pmatrix} \text{ʍ}^2{}_2^l+\text{Ⱥ}^2{}_2^l+\underline{\text{đ}}^2{}_2^l \\ +\text{ʍ}^2{}_2^u+\text{Ⱥ}^2{}_2^u+\underline{\text{đ}}^2{}_2^u \end{pmatrix}} \tag{16}$$

$$Iv-SFDSM^2(R_1,R_2)$$

$$=\frac{1}{k}\sum_{s=1}^{k}\ddot{W}_k \frac{2\begin{pmatrix} \text{ʍ}_1^l\text{ʍ}_2^l+\text{Ⱥ}_1^l\text{Ⱥ}_2^l \\ +\underline{\text{đ}}_1^l\underline{\text{đ}}_2^l+ \\ \text{r}_1^l\text{r}_2^l+\text{ʍ}_1^u\text{ʍ}_2^u+ \\ \text{Ⱥ}_1^u\text{Ⱥ}_2^u+\underline{\text{đ}}_1^u\underline{\text{đ}}_2^u+\text{r}_1^u\text{r}_2^u \end{pmatrix}}{\begin{pmatrix} \text{ʍ}^2{}_1^l+\text{Ⱥ}^2{}_1^l+\underline{\text{đ}}^2{}_1^l \\ +\text{r}^2{}_1^l+\text{ʍ}^2{}_1^u+ \\ \text{Ⱥ}^2{}_1^u+\underline{\text{đ}}^2{}_1^u+\text{r}^2{}_1^u \end{pmatrix}+\begin{pmatrix} \text{ʍ}^2{}_2^l+\text{Ⱥ}^2{}_2^l+\underline{\text{đ}}^2{}_2^l \\ +\text{r}^2{}_2^l+\text{ʍ}^2{}_2^u+ \\ \text{Ⱥ}^2{}_2^u+\underline{\text{đ}}^2{}_2^u+\text{r}^2{}_2^u \end{pmatrix}} \tag{17}$$

$$Iv-SFDSM^3(R_1,R_2)=$$

$$\frac{2\sum_{s=1}^{k}\ddot{W}_k\begin{pmatrix} \text{ʍ}_1^l\text{ʍ}_2^l+\text{Ⱥ}_1^l\text{Ⱥ}_2^l+\underline{\text{đ}}_1^l\underline{\text{đ}}_2^l+ \\ \text{ʍ}_1^u\text{ʍ}_2^u+\text{Ⱥ}_1^u\text{Ⱥ}_2^u+\underline{\text{đ}}_1^u\underline{\text{đ}}_2^u \end{pmatrix}}{\sum_{s=1}^{k}\ddot{W}_k\begin{pmatrix} \text{ʍ}^2{}_1^l+\text{Ⱥ}^2{}_1^l+\underline{\text{đ}}^2{}_1^l \\ +\text{ʍ}^2{}_1^u+\text{Ⱥ}^2{}_1^u+\underline{\text{đ}}^2{}_1^u \end{pmatrix}+\sum_{s=1}^{k}\ddot{W}_k\begin{pmatrix} \text{ʍ}^2{}_2^l+\text{Ⱥ}^2{}_2^l+\underline{\text{đ}}^2{}_2^l \\ +\text{ʍ}^2{}_2^u+\text{Ⱥ}^2{}_2^u+\underline{\text{đ}}^2{}_2^u \end{pmatrix}} \tag{18}$$

$$Iv-SFDSM^4(R_1,R_2)$$

$$=\frac{2\sum_{s=1}^{k}\ddot{W}_k\begin{pmatrix} \text{ʍ}_1^l\text{ʍ}_2^l+\text{Ⱥ}_1^l\text{Ⱥ}_2^l \\ +\underline{\text{đ}}_1^l\underline{\text{đ}}_2^l+ \\ \text{r}_1^l\text{r}_2^l+\text{ʍ}_1^u\text{ʍ}_2^u \\ +\text{Ⱥ}_1^u\text{Ⱥ}_2^u+\underline{\text{đ}}_1^u\underline{\text{đ}}_2^u+\text{r}_1^u\text{r}_2^u \end{pmatrix}}{\sum_{s=1}^{k}\ddot{W}_k\begin{pmatrix} \text{ʍ}^2{}_1^l+\text{Ⱥ}^2{}_1^l+\underline{\text{đ}}^2{}_1^l \\ +\text{r}^2{}_1^l+\text{ʍ}^2{}_1^u+ \\ \text{Ⱥ}^2{}_1^u+\underline{\text{đ}}^2{}_1^u+\text{r}^2{}_1^u \end{pmatrix}+\sum_{s=1}^{k}\ddot{W}_k\begin{pmatrix} \text{ʍ}^2{}_2^l+\text{Ⱥ}^2{}_2^l+\underline{\text{đ}}^2{}_2^l \\ +\text{r}^2{}_2^l+\text{ʍ}^2{}_2^u+ \\ \text{Ⱥ}^2{}_2^u+\underline{\text{đ}}^2{}_2^u+\text{r}^2{}_2^u \end{pmatrix}} \tag{19}$$

Theorem 5. Consider two Iv-SFSVs $R_1=\left([\text{ʍ}_1^l,\text{ʍ}_1^u],[\text{Ⱥ}_1^l,\text{Ⱥ}_1^u],[\underline{\text{đ}}_1^l,\underline{\text{đ}}_1^u]\right), R_2=\left([\text{ʍ}_2^l,\text{ʍ}_2^u],[\text{Ⱥ}_2^l,\text{Ⱥ}_2^u],[\underline{\text{đ}}_2^l,\underline{\text{đ}}_2^u]\right)$ and weight vectors $\sum_{s=1}^{k}\ddot{W}_k=1$ and $(\ddot{W}_1,\ddot{W}_2,\dots,\ddot{W}_k)\in[0,1]$, then holds the following axions for $P=1,2,3,4$.

i. $0\leq Iv-SFDWSM^P(R_1,R_2)\leq 1$
ii. $Iv-SFDWSM^P(R_1,R_2)=Iv-SFDWSM^P(R_2,R_1)$
iii. For $R_1=R_2, Iv-SFDWSM^P(R_1,R_2)=1$
iv. Let us assume the $R_1\subseteq R_2\subseteq R_3$, then $Iv-SFDSM^P(R_1,R_3)\leq Iv-SFDSM^P(R_1,R_2)\leq Iv-SFDSM^P(R_1,R_3)$.

***Proof*:**

i. As MV, AV, and NMV of all Iv-SFSVs belong to the closed interval $[0,1]$, then it is obvious that $Iv-SFDSM^P(R_1,R_2)\in[0,1]$.
ii. Satisfy trivially.
iii. If we take $R_1=R_2$ the is obviously $\text{ʍ}_1^l=\text{ʍ}_2^l, \text{Ⱥ}_1^l=\text{Ⱥ}_2^l, \underline{\text{đ}}_1^l=\underline{\text{đ}}_2^l, \text{ʍ}_1^u=\text{ʍ}_2^u, \text{Ⱥ}_1^u=\text{Ⱥ}_2^u, \underline{\text{đ}}_1^u=\underline{\text{đ}}_2^u$

Then

$$Iv-SFCWSM^1(R_1,R_2)=\sum_{s=1}^{k}\ddot{W}_k\frac{2\left(\text{ʍ}^2{}_1^l+\text{Ⱥ}^2{}_1^l+\underline{\text{đ}}^2{}_1^l+\text{ʍ}^2{}_1^u+\text{Ⱥ}^2{}_1^u+\underline{\text{đ}}^2{}_1^u\right)}{\left(\text{ʍ}^2{}_1^l+\text{Ⱥ}^2{}_1^l+\underline{\text{đ}}^2{}_1^l+\text{ʍ}^2{}_1^u+\text{Ⱥ}^2{}_1^u+\underline{\text{đ}}^2{}_1^u\right)+\left(\text{ʍ}^2{}_1^l+\text{Ⱥ}^2{}_1^l+\underline{\text{đ}}^2{}_1^l+\text{ʍ}^2{}_1^u+\text{Ⱥ}^2{}_1^u+\underline{\text{đ}}^2{}_1^u\right)}$$

$$Iv-SFCSM^1(R_1,R_2)=\sum_{s=1}^{k}\ddot{W}_k=1$$

4. Calculation of WV through analytical hierarchy process

The main objective of AHP is to calculate the weight vectors of attributes; first of all we need to define our goal, first stage 1. Then, evaluate the pairwise comparison at stage 2. Finally, calculate the weights of the attributes. The mechanism of weight calculations is stepwise given in Figure 1.

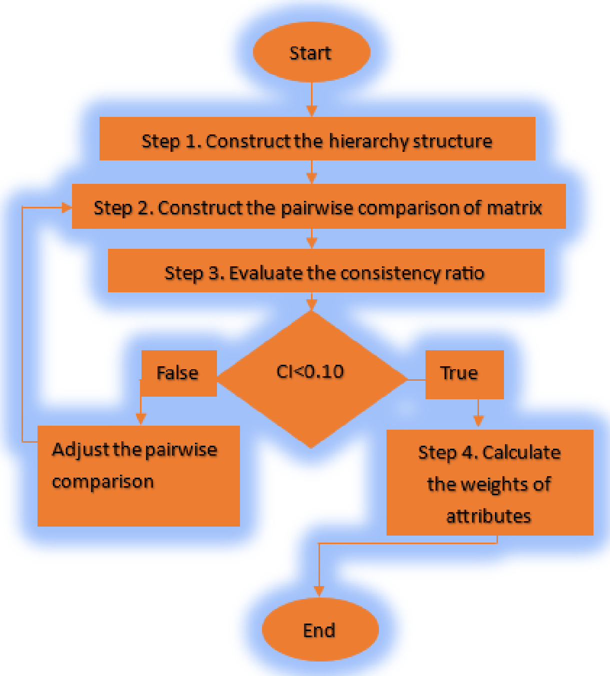

Fig. 1. Shows the calculation of weights of attributes through AHP

The flow graphic illustrates how we started by creating a matrix containing pairwise comparisons and linguistic measures. The fuzzy weight of characteristics is processed and determined by the AHP. We also assess the weights' consistency index (CI) under investigation. Weights are deemed accurate if the CI value is less than 0.1; if the CI value is greater than 0.1, the pairwise comparison is adjusted.

Table 1
Linguistic measure importance is used for pairwise comparison

Linguistic term	$([\text{ʍ}^{l}, \text{ʍ}^{u}], [\text{Ꞩ}^{l}, \text{Ꞩ}^{u}], [\text{đ}^{l}, \text{đ}^{u}])$	Scale indexing
Absolutly More important (AMI)	$\begin{pmatrix}[0.3, 0.2], [0.3, 0.1],\\ [0.3, 0.1], [0.1, 0.2]\end{pmatrix}$	9
Very high important (VHI)	$\begin{pmatrix}[0.8, 0.9], [0.1, 0.2],\\ [0.1, 0.4], [0.2, 0.5]\end{pmatrix}$	7
High important (HI)	$\begin{pmatrix}[0.8, 0.3], [0.4, 0.1],\\ [0.3, 0.4], [0.5, 0.9]\end{pmatrix}$	5
Slightly more critical (SMI)	$\begin{pmatrix}[0.3, 0.2], [0.9, 0.2],\\ [0.3, 0.1], [0.1, 0.2]\end{pmatrix}$	3
Equal importance (EI)	$\begin{pmatrix}[0.9, 0.5], [0,2, 0,4],\\ [0.3, 0.1], [0.1, 0.1]\end{pmatrix}$	1
Slightly low importance (SLI)	$\begin{pmatrix}[0.9, 0.2], [0.3, 0.2],\\ [0.3, 0.1], [0.1, 0.2]\end{pmatrix}$	1/3
Low importance (LI)	$\begin{pmatrix}[0.4, 0.1], [0.8, 0.3],\\ [0.3, 0.4], [0.5, 0.9]\end{pmatrix}$	1/5
Very low importance (VLI)	$\begin{pmatrix}[0.1, 0.2], [0.8, 0.9],\\ [0.1, 0.4], [0.2, 0.5]\end{pmatrix}$	1/7
Absolutly Low importance (ALI)	$\begin{pmatrix}[0.3, 0.1], [0.3, 0.2],\\ [0.3, 0.1], [0.1, 0.2]\end{pmatrix}$	1/9

4.1. Calculation of weight vectors

Step 1. Use linguistic scale values (Table 1) to create the decision matrix (DM) and compare each possibility pairwise.

$$B = (\mathrm{M})_{m\times n}$$

Where $(\mathrm{M})_{m\times n}$ Represents the relative significance of standards m to n.

Step 2. Investigate the geometric mean (Gm) Row-wise of the DM.

$$Gm_i = \prod_{i=1}^{\underline{\mathrm{n}}} \mathrm{M}_{m\times n}$$

Step 3. Compute the sum (s_i) of Gm.

$$s_i = \sum_{i=1}^{\underline{\mathrm{n}}} Gm_i$$

Step 4. Compute the inverse (I_i) of Gm.

$$I_i = (s)^{-1} = \frac{1}{\sum_{i=1}^{\underline{\mathrm{n}}} Gm_i}$$

Then, arrange $\left(I_1, I_2, \dots, I_{\underline{\mathrm{n}}}\right)$ in ascending order.

Step 5. Evaluate the product (p_i) of the s_i and arranged I_i.

$$p_i = \prod_{i=1}^{\underline{\mathrm{n}}} (s_i, I_i)$$

Step 5. Evaluate the average (a_i) of p_i.

$$a_i = \frac{\sum_{i=1}^{\underline{\mathrm{n}}} p_i}{\underline{\mathrm{n}}}$$

Where $\underline{\mathrm{n}} = 1, 2, \dots, i$.

Step 6. Dividing A_i with sum of $\sum_{i=1}^{\underline{\mathrm{n}}} A_i$ and obtain WVs by normalization of each priority weight.

$$\tilde{\varphi} = \frac{a_i}{\sum_{i=1}^{\underline{\mathrm{n}}} a_i}$$

Step 7. Examine the CI of computed $\tilde{\varphi}$ by using the following formula.

$$CI = \frac{\lambda_{\max} - 1}{\underline{\mathrm{n}} - 1}$$

Where,

$$\lambda_{max} = \sum_{i=1}^{\underline{\mathrm{n}}} a_i$$

and $\underline{\mathrm{n}}$ be the number of alternatives.

Step 8. Apply the developed similarity measure $Iv - SFWCSM^1, Iv - SFWCSM^2, Iv - SFWDSM^1, Iv - SFWDSM^2, Iv - SFWDSM^3, Iv - SFWDSM^4$.

5. Application

The CHD is a leading cause of death worldwide. Accurate diagnosis and early detection are critical to successful treatment and management. Computed Tomography Angiography (CTA) has developed as a useful non-invasive imaging method for detecting CHD by visualizing coronary arteries. However, manual interpretation of CTA pictures is time-consuming and prone to human mistakes. To address these problems, sophisticated image processing techniques, including fuzzy information measures and pattern recognition-based segmentation, have been developed to improve diagnosis accuracy and automate the localization of coronary artery blockages. Some major parts of the heart are shown in Figure 2.

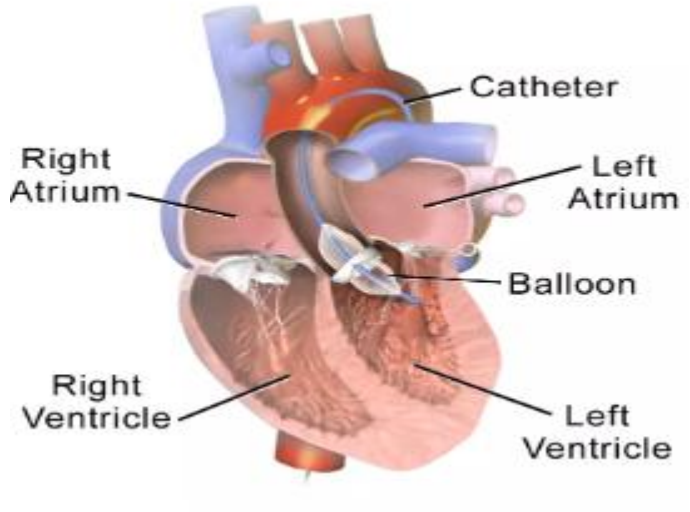

Fig. 2. Shows different parts of hearts

Timely detection of CHD is a significant and challenging problem for medical experts. This case study investigates using similarities measures based on segmentation approaches to improve CHD diagnosis using CTA pictures. By combining these new approaches, medical experts and doctors can detect coronary artery disease more accurately and quickly. Accurately identifying and localizing coronary artery regions impacted by blockages or narrowing is the main problem in diagnosing CHD using CTA. Traditional image processing techniques cannot manage the complexity and variety of CTA images because of problems, including noise in images, variations in patient anatomy, subtle obstructions, and time constraints. In medical imaging applications like CTA, where noise and fluctuations are frequent, SMs are especially well-suited since it is an effective method for managing uncertainty and imprecision in data. In heart emergency cases, the segmentation and localization in CTA play a crucial role. So, the assessment of CTA is based on attributes like accuracy of diagnosis, image quality, and segmentation quality shown in Figure 3.

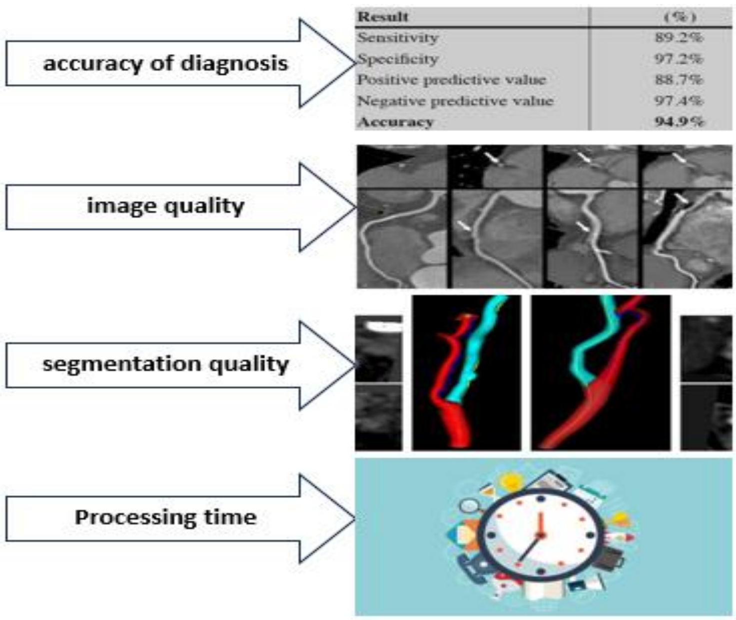

Fig. 3. Shows the attributes of the CTA

6. Numerical example

Consider we are selecting the best CTA report from the list of three reports. $\beta_i (i = 1, 2, 3)$ by comparing with normal reports β. For the selection process, we used proposed SMs $Iv - SFWCSM^1, Iv - SFWCSM^2, Iv - SFWDSM^1, Iv - SFWDSM^2, Iv - SFWDSM^3, Iv - SFWDSM^4$. Also, we have considered the following attributes: $\eth_1$ is the accuracy of diagnosis with weight 0.12, $\eth_2$ is the image quality with a weightage of 0.38, and $\eth_3$ is the segmentation quality with a weightage of 0.50. All weight vectors were calculated by AHP discussed above subsection. The arrangement of attributes and alternatives/options can be shown in Table 2.

Table 2
Decision matrix based on Iv-SFSVs

Options	β_1	β_2	β_3	β
$\eth_1$	([0.9, 0.5], [0.2, 0.4], [0.3, 0.1], [0.1, 0.1])	([0.1, 0.2], [0.8, 0.7], [0.1, 0.4], [0.2, 0.5])	([0.4, 0.1], [0.8, 0.3], [0.3, 0.4], [0.5, 0.9])	([0.5, 0.1], [0.3, 0.2], [0.2, 0.3], [0.4, 0.4])
$\eth_2$	([0.8, 0.7], [0.1, 0.2], [0.1, 0.4], [0.2, 0.5])	([0.9, 0.5], [0.2, 0.4], [0.3, 0.1], [0.1, 0.1])	([0.5, 0.2], [0.4, 0.3], [0.6, 0.6], [0.1, 0.9])	([0.4, 0.3], [0.3, 0.2], [0.9, 0.2], [0.2, 0.3])
$\eth_3$	([0.8, 0.], [0.4, 0.1], [0.3, 0.4], [0.5, 0.])	([0.3, 0.1], [0.2, 0.2], [0.3, 0.1], [0.1, 0.1])	([0.9, 0.5], [0.2, 0.4], [0.3, 0.1], [0.1, 0.1])	([0.4, 0.3], [0.8, 0.1], [0.4, 0.5], [0.3, 0.3])

We apply our proposed similarity measure to three different attributes β_1, β_2, β_3 and compare our results with β under the consideration of three attributes $\eth_1, \eth_2, \eth_3$ havi weigh vectors $(0.12, 0.38, 0.50)^T$. The aggregated outcomes are presented in Table 3.

Table 3
Aggregated results by using proposed SMs

SMs	(β_1, β)	(β_2, β)	(β_3, β)
$Iv - SFWCSM^1$	0.6171	0.3864	0.6715
$Iv - SFWCSM^2$	0.6559	0.3010	0.7188
$Iv - SFWDSM^1$	0.6118	0.3308	0.6916
$Iv - SFWDSM^2$	0.6737	0.3062	0.7613
$Iv - SFWDSM^3$	0.0625	0.0170	0.8239
$Iv - SFWDSM^4$	0.055	0.019	0.556

Table 2. above shows the aggregated results using six different proposed similarity measures. It is noticed that no SMs is the biggest in (β_1, β) and (β_2, β) while six SMs are the biggest in (β_3, β) have the biggest value among all SMs. So, we finally declared that the alternative β_3 The best option among all considered options. The graphical representation of the proposed work is presented in Figure 4.

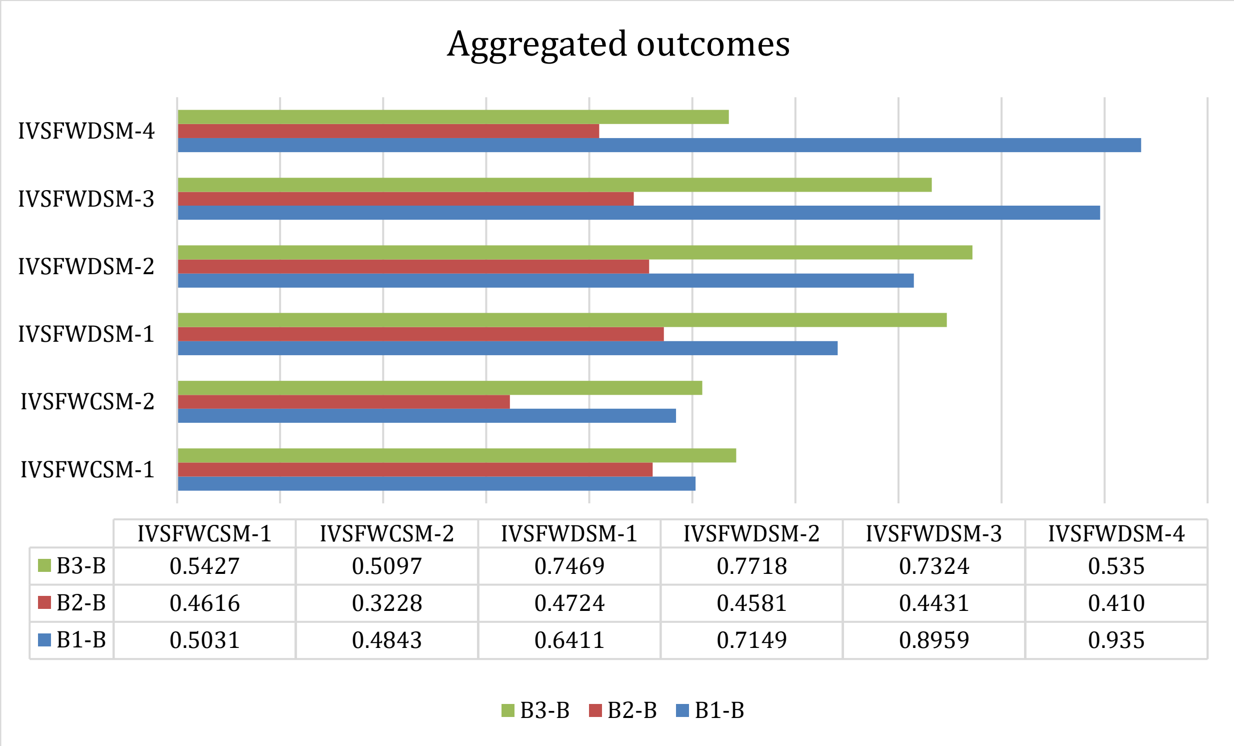

Figure 4. Geometrical representation of aggregated results by using proposed SMs

7. Comparative analysis

The main theme of this section is to show the superiority of the proposed methodology and its effectiveness. The proposed theory is also the generalization of interval-valued PFS. So, we compare developed SMs with interval-valued PFS theory-based SMs discussed by Liu *et al.*, [20]. It is also noticed that during the comparison process, many FS theory extensions like FS, IFS, and Iv-FS cannot handle Iv-SFS based SMs due to deficiency in their structure. The aggregated outcomes are presented in Table 4.

Table 4
Aggregated results by using proposed SMs

SMs	(β_1, β)	(β_2, β)	(β_3, β)
$Iv-PFWCSM^1$	0.5031	0.4616	0.5427
$Iv-PFWCSM^2$	0.4843	0.3228	0.5097
$Iv-PFWDSM^1$	0.6411	0.4724	0.7469
$Iv-PFWDSM^2$	0.7149	0.4581	0.7718
$Iv-PFWDSM^3$	0.8959	0.4431	0.7324
$Iv-PFWDSM^4$	0.935	0.410	0.535

Table 4 presents the aggregated outcomes usissssng six proposed similarity measures. It is observed that two SMs are the biggest in (β_1, β) and (β_2, β) four SMs are the biggest in (β_3, β) have the biggest value among all SMs. So, we finally say that the alternative β_3 The best alternative among all considered alternatives.

8. Conclusion

In medical sciences, the thought of SMs is a wonderful approach where MADM is involved. The Iv-SFS generalizes interval-valued FS, IFS, and PFS and can aggregate information precisely where ordinary FS, IFS, and PFS failed. By using the framework of Iv-SFS, we developed a new family of SMs like $Iv - SFWCSM^1, Iv - SFWCSM^2, Iv - SFWDSM^1, Iv - SFWDSM^2, Iv - SFWDSM^3, Iv - SFWDSM^4$ including basic axioms of SMs. Using these proposed SMs, we have investigated the assessment mechanism CTA for the detection of CHD. We have discussed case studies on CHD under attributes like assessing the best CTA report. For better illustration, we offered a numerical example for investigating the CTA report. To verify the effectiveness and applicability of the proposed approach, we made a comparison with the present approach and discussed some advantages of the proposed approach.

We aim to extend our developed theory to different fuzzy frameworks and operational laws in the future. For example, the concept of bipolar FS was given by Mahmood [26], and Rehman and Mahmood [27] discussed the thought of a Picture fuzzy N-soft set. The idea of Fermatean FS was presented by Senapati and Yager [28].

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Conflicts of Interest

The authors declare no conflicts of interest.

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